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Fiscal Policy as a Stabilizing Tool

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Evolution of Financial Institutions: Iceland's Path from Repression to Eruption

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Fiscal Policy as a Stabilizing Tool

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Abstract

One of the main focuses in the economic policy literature over the last decades has been on the role of monetary policy in stabilizing business cycles. This paper is concerned with the role of fiscal policy as a stabilizing tool and the effect of this on monetary policy. A Neo-Keynesian model is used to analyze how different fiscal policy rules affect the business cycle. The results indicate that using a counter-cyclical government spending policy along with progressive taxation is the most successful policy in mitigating fluctuations. The second most successful policy is to use a counter-cyclical spending policy along with a constant tax rate. However, using these policies both requires a lot of information and is difficult to implement with respect to timing. Hence, it can result in more fluctuations in output than expected. It may therefore be feasible to implement our third-best fiscal rule, i.e., fix the tax rate and allow government spending to grow annually by a predetermined rate equal to the growth rate of potential output.

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Introduction

It is well-known that people choose to smooth consumption over their life cycle (Ando and Modigliani, 1963). People borrow when they are young, pay off their debts and save during their middle age, and withdraw savings when retired. Consumption smoothing, and the fact that capital markets are imperfect, is the main reason for governments implementing economic policy. People gain more welfare when there is less fluctuation in production and consequently in consumption (see Storresletten, Telmer and Yaron, 2001). Also, high inflation as a consequence of bad or no policy can affect production in the long run and thereby income (see Herbertsson and Gylfason, 2001). This result further strengthens the view that government has a role to play in macroeconomic policy.

By looking at Table 1, it is easy to conclude that the industrialized countries have not been very successful in implementing an effective counter-cyclical fiscal policy during the last decades. If a country has been implementing a successful counter-cyclical fiscal policy, a negative correlation should arise between government consumption and the output gap.² However, this only seems to be the case in four out of 26 OECD countries: the UK, France, Austria and Sweden.

Table 1. Correlation between government consumption and output in 26 OECD countries in 1971 - 2001

<i>Deviations from HP-trend</i>			
Country	Correlation	Country	Correlation
Luxembourg*	0.43	Japan	0.16
Turkey*	0.42	Korea	0.15
Italy*	0.37	Denmark	0.05
Portugal*	0.30	Iceland	0.03
Ireland*	0.28	Netherlands	-0.10
United States*	0.28	New Zealand	-0.11
Australia*	0.28	Norway	-0.15
Spain*	0.27	Canada	-0.17
Greece	0.26	Germany	-0.20
Switzerland	0.21	United Kingdom*	-0.27
Belgium	0.20	France*	-0.30
Finland	0.19	Austria*	-0.30
Mexico	0.18	Sweden*	-0.46

*Statistically significant

² Note that this table only shows correlation of the output gap with government consumption. One could also show the correlation with government spending or government budget deficit to conclude on how countries have been doing their fiscal policies.

There can be several reasons for this result. One is simply that the countries were not following counter-cyclical fiscal policies, but if indeed they were following such a policy, the reason for its lack of success may be wrong timing of action. It takes time to make and implement fiscal policy, and there is little information about where a country is in the business cycle, except ex post.

This has resulted in economists abandoning the idea of discretionary counter-cyclical fiscal policy. Another reason for this development is that such a policy is much more inflexible than a monetary policy, and it is easier to make a monetary policy credible (see Friedman, 1968, Persson and Tabellini, 1990 and 2000, Taylor, 1993). Further, monetary policy has changed significantly over the last 15 years: Interest rates are currently the main tool for fighting inflation; exchange-rate determination has been liberalized, and restraints on capital movements between countries have been lifted (see Mishkin, 1999). Interest-rate determinations have become more explicit, systematic and more responsive to changes in the inflation rate and production. These changes have led to less economic fluctuation in the U.S., which makes it less likely that a discretionary fiscal policy would lead to a better economic outcome. Active discretionary fiscal policy could even make an effective monetary policy more difficult. All these factors increase the need for a systematic and explicit fiscal policy. Discretionary fiscal policy should be concerned with long-term issues, like reforms in tax and social insurance issues, but not the business cycle (see Taylor 2000).

In this paper we use a stochastic, Neo-Keynesian general equilibrium model to compare different settings in determining government consumption and taxes and the effects that these different settings have on monetary policy and the business cycle. This model is similar to other Neo-Keynesian general equilibrium model that have appeared in the literature (see for example Walsh (2003), ch. 5, for an overview of the closed economy case and Krugman and Obstfeld (1995) for an example of it's open economy counterpart). However, there is one main difference, namely that we assume relative income taxes in the economy instead of only lump sum taxation. The difference allows us to draw richer conclusions concerning the effects of different fiscal policies on economic fluctuations, monetary policy and inflation. Further, we solve our model analytically using log-linear approximations and use parameter

values, both that are known in the literature and that we calculate, to be able to draw clear-cut conclusions from the model.

I. The Model Economy

In this section we develop a Neo-Keynesian stochastic general equilibrium model with a public sector. The model is different from the traditional RBC model in four ways. The first is that the capital stock is assumed to be fixed. This is justified by the fact that there seems to be little relationship between the capital stock and output at business cycle frequencies in the United States as is shown by McCallum and Nelson (1999). Also, as Cogley and Nason (1995) show, investment and capital respond little to productivity shocks, which is one of two driving forces of fluctuations in our model. The second difference is that we assume differentiated goods whose prices are set by monopolistically competitive firms facing price stickiness as in Calvo (1983), where there is a certain probability that a given firm will change the price of its output in a given time period. The third difference is that we assume that there are shocks to preferences. This is done to incorporate shocks to demand, as well as supply, in the model. Finally, monetary policy is represented by a rule for setting the nominal interest rate level and the nominal quantity of money is determined by the interest rate level, and not vice versa.

The model economy consists of identical and infinitely lived households, firms, a government and a central bank. The households consume private and public goods, supply labor, hold government debt and money balances, which they can use to buy consumption goods. The firms produce private and public goods that they sell to households and the government. The government buys public consumption goods, raises taxes and issues bonds to finance its spending. The central bank distributes money balances to the households that can be used to purchase consumption goods.

1.1. Households

The representative household derives utility from consumption (C), money holding (M/P) and leisure ($1-N$).³ Hence, the expected net present value of lifetime utility at time t for the representative household can be written as:

³ Total time available is normalized to 1.

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(\psi_{t+i} C_{t+i})^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right] \quad (1.1)$$

where E_t is the mathematical expectations operator given the available information at time t , $\beta \in (0,1)$ is a subjective discount factor, reflecting the time preference of the representative household,⁴ ψ is a exogenously determined taste shock,⁵ that has average of 1, and σ , b , γ , χ and η are positive parameters. C is composite consumption defined as:

$$C_t = \left[\int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (1.2)$$

where c_j is consumption of good j and θ is a parameter ($\theta > 1$).⁶

The household's maximization problem can be dealt with in two steps. First, the household chooses the quantity it consumes of individual consumption goods by solving:

$$\min_{c_{jt}} \int_{j=0}^1 p_{jt} c_{jt} dj \quad (1.3)$$

subject to equation (1.2). This results in the following Marshallian demand equation for each good:⁷

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t \quad (1.4)$$

where the aggregate price level is defined as: $P_t \equiv \left[\int_{j=0}^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$.⁸ Second, the

household maximizes its lifetime utility in (1.2) by choosing C , N , M and B subject to the following budget constraint:

$$\begin{aligned} C_{t+i} + \frac{M_{t+i}}{P_{t+i}} + \frac{B_{t+i}}{P_{t+i}} \\ = \left(\frac{W_{t+i}}{P_{t+i}} \right) (1 - \tau_{t+i}) N_{t+i} + \frac{M_{t+i-1}}{P_{t+i}} + (1 + i_{t+i-1}) \left(\frac{B_{t+i-1}}{P_{t+i}} \right) + \Pi_{t+i} \end{aligned} \quad (1.5)$$

⁴ β equals the inverse of 1 + the households subjective interest rate.

⁵ We will define this process in chapter 2.5.

⁶ In setting up the model we partly follow Walsh (2003), Chapter 5.

⁷ This result is derived in Appendix A.1.

⁸ This is the marginal cost of increasing aggregate consumption by one composite unit.

where M is the nominal money balance, B are holdings of government bonds, W are wages, Π is real profit from firms received by the household and τ is labor tax rate.⁹

Solving the households maximization problem results in the following first order conditions:¹⁰

$$\frac{W_t}{P_t}(1 - \tau_t) = \frac{\chi N_t^\eta}{\psi_t^{1-\sigma} C_t^{-\sigma}} \quad (1.6)$$

$$\frac{\psi_t^{1-\sigma} C_t^{-\sigma}}{P_t} = \beta(1 + i_t) E_t \left(\frac{\psi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma}}{P_{t+1}} \right) \quad (1.7)$$

$$\frac{\gamma \left(\frac{M_t}{P_t} \right)^{-b}}{\psi_t^{1-\sigma} C_t^{-\sigma}} = \frac{i_t}{1 + i_t} \quad (1.8)$$

Equation 1.6 - 1.8 along with the budget constraint in (1.5) can be used to derive the household's optimal paths for C , N , M and B .

1.2. Market Equilibrium

To ensure market equilibrium the following has to hold for every good j :

$$y_{jt} = c_{jt} + g_{jt} \quad (1.9)$$

where y_j is output of good j and g_j is government purchases of good j . The ratio between government and private purchases is assumed to be identical for all goods.¹¹ This can be justified by the fact that we do not distinguish between products in the economy, i.e. they all have the same level of substitutability (same θ) and hence this should not be an unreasonable assumption. Hence, we can write this ratio independent

of the goods, i.e. $\alpha_t = \frac{g_{jt}}{c_{jt}}$, and equation (1.9) in the following way:

$$y_{jt} = (1 + \alpha_t) c_{jt} \quad (1.10)$$

⁹ Since distortions (taxes etc.) are present in the model solving it via a social planner gives different results than the market solution. Hence we solve the maximization problem for each agent in the model, i.e. the household and the firm, to derive the solution paths for the variables.

¹⁰ These are derived in Appendix A. 2.

¹¹ This assumption is similar to the ones made in the literature concerning government consumption. For example, in Obstfeld and Rogoff (1995), it is assumed that the government's demand function for a good is identical to the household's demand function for the good in the way that the share of private demand for a particular good in total consumption and the share of government demand for a good in total government consumption are equal. Like the assumption made her, this results in a fixed proportion between total private consumption (C) and total government consumption (G) at every point in time (α).

Using the fact that composite production can be written as $Y_t = \left[\int_{j=0}^1 y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$, then

total production can be written as:¹²

$$Y_t = (1 + \alpha_t)C_t = C_t + G_t \quad (1.11)$$

1.3. Firms

The firms operate in monopolistic competitive output markets. They set prices as to maximize profits every period. Following Calvo (1983), some firms choose to change their price in a given period and some not. The probability that a firm changes the price of its good is $(1-\omega)$ and the probability that it does not is ω . The degree of nominal rigidity is indicated by the value of $\omega < 1$.

The production technology for firm j is given by:

$$y_{jt} = Z_t N_{jt} \quad (1.12)$$

where Z is a stochastic process which determines labor productivity and has an average of 1.¹³ The problem with choosing the amount of labor the firms use every time period can be written as:

$$\min_{N_{jt}} \left(\frac{W_t}{P_t} \right) N_{jt} \quad (1.13)$$

subject to (1.12). This results in the following marginal cost of production for the firms (mc):¹⁴

$$mc_t = \frac{W_t / P_t}{Z_t} \quad (1.14)$$

The firms that do adjust their price in period t do so to maximize the expected discounted current and future profits. Each firm's problem is therefore to choose $p_{j,t}$ such that the following is maximized:

$$E_t \sum_{i=0}^{\infty} \omega^i \tilde{\beta}_{t,t+i} \left[\left(\frac{p_{j,t}}{P_{t+i}} \right) y_{j,t+i} - mc_{t+i} y_{j,t+i} \right] \quad (1.15)$$

¹² This result is derived in Appendix A.3.

¹³ We will define this process in chapter 2.5.

¹⁴ These results are derived in Appendix A.4.

where $\tilde{\beta}_{t,t+i} = \beta^i \frac{\psi_{t+i}^{1-\sigma} C_{t+i}^{-\sigma}}{\psi_t^{1-\sigma} C_t^{-\sigma}}$ is the discount factor (obtained from equation (1.7)). The

first order condition for the firm's maximization problems is the following after

substituting $\tilde{\beta}_{t,t+i} = \beta^i \frac{\psi_{t+i}^{1-\sigma} C_{t+i}^{-\sigma}}{\psi_t^{1-\sigma} C_t^{-\sigma}}$ and equation (1.11) into the first order condition:¹⁵

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left[(1-\theta) \left(\frac{P_t^*}{P_{t+i}} \right) + \theta mc_{t+i} \right] \left(\frac{1}{P_t^*} \right) \left(\frac{P_t^*}{P_{t+i}} \right)^{-\theta} \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^\sigma = 0 \quad (1.16)$$

where p^* is the optimal price. This condition gives the optimal price for the goods whose prices are changed in time t (p^*). Using also that the price level index

$P_t \equiv \left[\int_{j=0}^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$ can be rewritten as:¹⁶

$$P_t = \left[(1-\omega) p_t^{*1-\theta} + \omega P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (1.17)$$

This enables us to use (1.15) and (1.16) to determine that aggregate price level (P) in the economy.

1.4. Government

The government finances its purchases of final consumption goods by issuing government debt (bonds and money) and raising taxes. Hence its budget constraint can be written as:

$$\frac{M_{t+i-1}}{P_{t+i}} + (1+i_{t+i-1}) \frac{B_{t+i-1}}{P_{t+i}} + G_{t+i} = \frac{M_{t+i}}{P_{t+i}} + \frac{B_{t+i}}{P_{t+i}} + \frac{W_{t+i}}{P_{t+i}} \tau_{t+i} N_{t+i} \quad (1.18)$$

The budget constraint states that the government's end of period debt plus tax revenues during the period have to be equal to the beginning of period debt plus interest rate payment on debt during the period plus government consumption.

We will start by letting the government's decision concerning the tax rate and its purchases of final consumption goods be a stochastic process.¹⁷ Then we will try different arrangement in determining government consumption and taxes for comparison.

¹⁵ This result is derived in Appendix A.5.

¹⁶ Remember that ω is the probability that a price does not change in a given period.

¹⁷ This is discussed further in chapter 2.3.

1.5. Central Bank

The central bank's policy is to set interest rates in the economy to achieve their inflation and production targets, i.e. it is assumed to follow a Taylor rule:

$$i_t = r^* + \pi_t + \lambda_y (y_t - y^e) + \lambda_\pi (\pi_t - \pi^T) \quad (1.19)$$

where $y = \ln(Y)$, r^* is the equilibrium real interest rate π is the inflation rate ($\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$), π^T is the target inflation rate which is assumed to equal the steady

state inflation rate of zero, y^e is equilibrium production level (in logs) and λ_y and λ_π are coefficients. The money stock is then allowed to adapt to the interest rate level according to the demand for money resulting from the household's optimization problem (the condition in equation 1.8).

2. The Model Solved

In this section we obtain paths for inflation, interest rates, and the output gap and compare with different determination of government consumption and taxes. We will therefore start by minimizing the model with respect to that objective.

2.1. The Demand Side (The Forward Looking IS Curve)

Linearizing equation 1.7 around the variables' zero inflation steady states gives the first order stochastic difference equation for production in the economy:¹⁸

$$\hat{y}_t = E_t \hat{y}_{t+1} - \gamma_1 (\hat{i}_t - E_t \hat{\pi}_{t+1}) - \gamma_2 (E_t \hat{g}_{t+1} - \hat{g}_t) - \gamma_3 (E_t \hat{\psi}_{t+1} - \hat{\psi}_t) \quad (2.1)$$

where:

$\gamma_1 = \frac{C^{SS}}{Y^{SS}} \frac{1}{\sigma}$, $\gamma_2 = \frac{G^{SS}}{Y^{SS}}$, $\gamma_3 = \frac{C^{SS}}{Y^{SS}} \frac{1-\sigma}{\sigma}$, $\hat{y} = Y/Y^{SS} - 1$, $\hat{i} = \beta(1+i) - 1 \approx i - i^{SS}$, $\hat{\pi} = \pi$, $\hat{g} = G/G^{SS} - 1$, $\hat{\psi} = \Psi/1 - 1$ and Y^{SS} , C^{SS} , G^{SS} are the steady state values of Y , C and G .¹⁹ This equation represents the demand side of the economy (forward looking IS curve). Note that we also have the following information concerning the

¹⁸ This equation is derived in Appendix A.6.

¹⁹ Remember that the average of the taste shock, and hence its steady state value, is 1 and that we assume a steady state inflation rate of zero.

parameters in (2.1): $\gamma_1 > 0$, $\gamma_2 > 0$ since $0 < \frac{C^{SS}}{Y^{SS}} < 1$, $0 < \frac{G^{SS}}{Y^{SS}} < 1$ and $\sigma > 0$. Also, $\gamma_3 > 0$, $\gamma_3 < 0$, $\gamma_3 = 0$ if $\sigma < 1$, $\sigma > 1$, $\sigma = 0$.

2.2. The Supply Side (The Neo-Keynesian Philips Curve)

Linearizing equation (1.16) around the variables' zero inflation steady states gives the first order stochastic difference equation for inflation in the economy:²⁰

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{m}c_t \quad (2.2)$$

where $\kappa \equiv \frac{(1-\omega\beta)(1-\omega)}{\omega}$, $\hat{m}c = mc/mc^{SS} - 1$ and mc^{SS} is the steady state value of mc . If we go one step further and plug in for $\hat{m}c$, equation (2.2) becomes:²¹

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_1 \hat{y}_t - \kappa_2 \hat{g}_t - \kappa_3 \hat{\psi}_t - \kappa_4 \hat{z}_t + \kappa_5 \hat{\tau}_t \quad (2.3)$$

where $\hat{\tau} = \frac{\tau - \tau^{SS}}{1 - \tau^{SS}} \approx \tau - \tau^{SS}$ and τ^{SS} is the steady state tax rate,

$$\kappa_1 \equiv \frac{(1-\omega\beta)(1-\omega)(\eta\alpha + \sigma)}{\omega\alpha}, \quad \kappa_2 \equiv \frac{\sigma\delta(1-\omega\beta)(1-\omega)}{\omega\alpha},$$

$$\kappa_3 \equiv \frac{(1-\sigma)(1-\omega\beta)(1-\omega)}{\omega}, \quad \kappa_4 \equiv \frac{(1+\eta)(1-\omega\beta)(1-\omega)}{\omega}, \quad \kappa_5 \equiv \frac{(1-\omega\beta)(1-\omega)}{\omega}$$

$\alpha = \frac{C^{SS}}{Y^{SS}}$ and $\delta = \frac{G^{SS}}{Y^{SS}}$. Note that $\kappa_1 > 0$, $\kappa_2 > 0$, $\kappa_4 > 0$ and $\kappa_5 > 0$ since

$0 < \beta < 1$, $0 < \omega < 1$, $\eta > 0$, $0 < \alpha < 1$, $\sigma > 0$ and $0 < \delta < 1$. Also, $\kappa_3 > 0$, $\kappa_3 < 0$, $\kappa_3 = 0$ if $\sigma < 1$, $\sigma > 1$, $\sigma = 0$.

2.3. Government Consumption

We will try four settings of government consumption and tax determination. In the first scenario we assume that deviations in government consumption and tax rates from their steady state values are fixed, i.e. we assume that government consumption is a trend process and that the tax rate is fixed. In the second setting we assume that deviation of government consumption from its steady state is a completely random process and that the tax rate is fixed. In the third setting we assume that there is a correlation between deviations in government consumption from its steady state value

²⁰ This equation is derived in Appendix A.7.

²¹ This equation is derived in Appendix A.10.

and the output gap and that the tax rate is fixed. In setting four, we assume the same setting as in three except that deviation in the tax rate from its steady state value is now correlated with the output gap. Hence, these settings are:

$$1. \quad \hat{g}_t = \hat{\tau}_t = 0 \quad (2.4)$$

$$2. \quad \hat{g}_t \sim iid(0, \sigma_g^2), \quad \hat{\tau}_t = 0 \quad (2.5)$$

$$3. \quad \hat{g}_t = \phi \hat{y}_t, \quad \hat{\tau}_t = 0 \quad (2.6)$$

$$4. \quad \hat{g}_t = \phi \hat{y}_t, \quad \hat{\tau}_t = \theta \hat{y}_t \quad (2.7)$$

2.4. Interest Rate Determination

The equilibrium, or steady state, real interest rate (r^*) is such that $\beta = \frac{1}{1+r^*}$ since the equilibrium inflation rate is zero. Hence, we can approximate and set $\hat{i} = i - i^{SS} = i - r^*$. Also, noting that $\hat{y} = y - y^e$ we can write equation (1.19) in the following way:

$$\hat{i}_t = \lambda_y \hat{y}_t + (1 + \lambda_\pi) \hat{\pi}_t \equiv \lambda_y \hat{y}_t + \lambda_\pi^* \hat{\pi}_t \quad (2.8)$$

Equation (2.8) describes interest rate determination in the economy.

2.5. Preferences and Labor Productivity

The random processes for taste and productivity are assumed to be independently and identically distributed:

$$\hat{\psi}_t \sim iid(0, \sigma_\psi^2) \quad (2.9)$$

$$\hat{z}_t \sim iid(0, \sigma_z^2) \quad (2.10)$$

Note that the shocks are assumed to be independent of each other.

2.6. The Model Summed Up

The minimized model consists of seven variables (\hat{y} , \hat{i} , $\hat{\pi}$, \hat{g} , $\hat{\tau}$, $\hat{\psi}$ and \hat{z}) and seven equations (2.1, 2.3, 2.4 or 2.5 or 2.6 or 2.7, 2.8, 2.9 and 2.10).²² The last two variables, i.e. the taste- and productivity shocks, are exogenous to the model and equations (2.9) and (2.10) describe their generating process. Deviation in government consumption from its steady state value is only exogenous in settings 1 and 2 in which

²² Remember that each of (2.4)-(2.7) consists of two equations.

(2.4) or (2.5) describe its generating process. In settings 3 and 4 it is endogenous and (2.6) or (2.7) is used to describe its generating process. The deviation in the tax rate from its steady state value is exogenous in settings 1, 2 and 3 in which (2.4) or (2.5) or (2.6) describe its generating process. In setting 4 it is endogenous and 2.7 is used to describe it.

2.7. The Model Solved

First we find a general solution for the model, i.e. we only assume that \hat{y} , $\hat{\pi}$, \hat{i} are endogenous and that $\hat{\tau}$, \hat{g} , $\hat{\psi}$, \hat{z} are exogenous stochastic processes whose expected values are 0. Hence, we guess for a solution for \hat{y} , $\hat{\pi}$ and \hat{i} of the following form:

$$\hat{y}_t = b_1 \hat{g}_t + b_2 \hat{\tau}_t + b_3 \hat{\psi}_t + b_4 \hat{z}_t \quad (2.11)$$

$$\hat{\pi}_t = c_1 \hat{g}_t + c_2 \hat{\tau}_t + c_3 \hat{\psi}_t + c_4 \hat{z}_t \quad (2.12)$$

$$\hat{i}_t = d_1 \hat{g}_t + d_2 \hat{\tau}_t + d_3 \hat{\psi}_t + d_4 \hat{z}_t \quad (2.13)$$

Where the b 's, c 's and d 's are parameters. This gives the following results for the parameters of (2.11) and (2.12):²³

$$b_1 = \frac{\gamma_1 \lambda_\pi^* \kappa_2 + \gamma_2}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \quad (2.14)$$

$$c_1 = \frac{\kappa_1 \gamma_2 - \kappa_2 (1 + \gamma_1 \lambda_y)}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)}$$

$$b_2 = -\frac{\kappa_5 \gamma_1 \lambda_\pi^*}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \quad (2.15)$$

$$c_2 = \frac{\kappa_5 (1 + \gamma_1 \lambda_y)}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)}$$

$$b_3 = \frac{\gamma_3 + \kappa_3 \gamma_1 \lambda_\pi^*}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \quad (2.16)$$

$$c_3 = \frac{\kappa_1 \gamma_3 - \kappa_3 (1 + \gamma_1 \lambda_y)}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)}$$

$$b_4 = \frac{\kappa_4 \gamma_1 \lambda_\pi^*}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \quad (2.17)$$

$$c_4 = -\frac{\kappa_4 (1 + \gamma_1 \lambda_y)}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)}$$

²³ These results are derived in Appendix A.11.

Obtaining values for the parameters of the model, calculating $b_1, b_2, b_3, b_4, c_1, c_2, c_3$ and c_4 and plugging these into (2.11) and (2.12) gives the developments of \hat{y} and $\hat{\pi}$. Using (2.8) gives development of \hat{i} where: $d_1 = \lambda_y b_1 + \lambda_\pi^* c_1$, $d_2 = \lambda_y b_2 + \lambda_\pi^* c_2$, $d_3 = \lambda_y b_3 + \lambda_\pi^* c_3$, $d_4 = \lambda_y b_4 + \lambda_\pi^* c_4$.

2.7.1. The Model Solved for Setting 1

Now the parameter values in (2.11) and (2.12) are the same as before. We only have to add the conditions in (2.4) to equations (2.11) and (2.12) to get the results:

$$\hat{y}_t = b_3 \hat{\psi}_t + b_4 \hat{z}_t \quad (2.18)$$

$$\hat{\pi}_t = c_3 \hat{\psi}_t + c_4 \hat{z}_t \quad (2.19)$$

The parameter values are given in (2.16) and (2.17). Then using (2.8) gives development for \hat{i} as before.

2.7.2. The Model Solved for Setting 2

Now the parameter values in (2.11) and (2.12) are the same as before. We only have to add the conditions in (2.5) to equations (2.11) and (2.12) to get the results:

$$\hat{y}_t = b_1 \hat{g}_t + b_3 \hat{\psi}_t + b_4 \hat{z}_t \quad (2.20)$$

$$\hat{\pi}_t = c_1 \hat{g}_t + c_3 \hat{\psi}_t + c_4 \hat{z}_t \quad (2.21)$$

The parameter values are given in (2.14), (2.16) and (2.17). Then using (2.8) gives development for \hat{i} as before.

2.7.3. The Model Solved for Setting 3

Using the condition in (2.6) we guess a solution for \hat{y} , $\hat{\pi}$ and \hat{i} of the following form:

$$\hat{y}_t = b_3 \hat{\psi}_t + b_4 \hat{z}_t \quad (2.22)$$

$$\hat{\pi}_t = c_3 \hat{\psi}_t + c_4 \hat{z}_t \quad (2.23)$$

$$\hat{i}_t = d_3 \hat{\psi}_t + d_4 \hat{z}_t \quad (2.24)$$

This gives the following results for the parameters of (2.22) and (2.23):²⁴

²⁴ These results are derived in Appendix A.12.

$$\begin{aligned}
b_3 &= \frac{\gamma_1 \lambda_\pi^* \kappa_3 + \gamma_3}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi) \lambda_\pi^*) - \gamma_2 \phi} \\
c_3 &= \frac{(\kappa_1 - \kappa_2 \phi) \gamma_3 - \kappa_3 (1 + \gamma_1 \lambda_y - \gamma_2 \phi)}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi) \lambda_\pi^*) - \gamma_2 \phi}
\end{aligned} \tag{2.25}$$

$$\begin{aligned}
b_4 &= \frac{\gamma_1 \lambda_\pi^* \kappa_4}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi) \lambda_\pi^*) - \gamma_2 \phi} \\
c_4 &= -\frac{\kappa_4 (1 + \gamma_1 \lambda_y - \gamma_2 \phi)}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi) \lambda_\pi^*) - \gamma_2 \phi}
\end{aligned} \tag{2.26}$$

Then using (2.8) gives development for \hat{i} as before.

2.7.4. The Model Solved for Setting 4

Using the condition in (2.7) we guess a solution for \hat{y} , $\hat{\pi}$ and \hat{i} as in (2.22), (2.23), and (2.24). This gives the following results for the parameters of (2.22) and (2.23):²⁵

$$\begin{aligned}
b_3 &= \frac{\gamma_1 \lambda_\pi^* \kappa_3 + \gamma_3}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \lambda_\pi^*) - \gamma_2 \phi} \\
c_3 &= \frac{(\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \gamma_3 - \kappa_3 (1 + \gamma_1 \lambda_y - \gamma_2 \phi)}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \lambda_\pi^*) - \gamma_2 \phi}
\end{aligned} \tag{2.27}$$

$$\begin{aligned}
b_4 &= \frac{\gamma_1 \lambda_\pi^* \kappa_4}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \lambda_\pi^*) - \gamma_2 \phi} \\
c_4 &= -\frac{\kappa_4 (1 + \gamma_1 \lambda_y - \gamma_2 \phi)}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \lambda_\pi^*) - \gamma_2 \phi}
\end{aligned} \tag{2.28}$$

Then using (1.8) gives development for \hat{i} as before.

3. Results

In this section we parameterize the model and calculate the b 's, c 's and d 's in shown in Section 2.7. Then we will be able to analyse the effects of each exogenous variables on the endogenous variables as well as to discuss and compare the effects of the four setting in determining government consumption and taxes.

3.1. Parameter Values

The parameter values were obtained from various sources. Their values and references to the sources are listed in table 2 below:

²⁵ These results are derived in Appendix A.13.

Parameter	β	σ	ω	η	α	δ	λ_π	λ_y
Value	0.99	0.5, 1, 2	0.8	1	0.83	0.17	0.5	0.5
Reference	Walsh (2003)	Authors	Walsh (2003)	Authors	Authors	Authors	Walsh (2003)	

Note: (i) $\alpha = C^{ss}/Y^{ss}$, $\delta = G^{ss}/Y^{ss}$ and $Y^{ss} = C^{ss} + G^{ss}$ and (ii) $\lambda_\pi^* = 1 + \lambda_\pi$.

The frequency of the model is a quarterly one as can be seen from the value of β . The value of σ is usually chosen equal to 1 as in Walsh (2003). However, in Herbertsson (2003) σ was estimated to equal 0.5 using data for the Nordic countries. Here, we will report results for $\sigma = 0.5$, $\sigma = 1$ and $\sigma = 2$ to show that the results concerning the effects of different fiscal policies are robust with respect to the values chosen for σ . The values for α and δ were obtained using data for USA in 1960 – 2003.

The following table shows the calculated parameters for equations (2.1) and (2.3) using the parameter in table 3:

Parameter	γ_1	γ_2	γ_3	κ_1	κ_2	κ_3	κ_4	κ_5
$\sigma = 0.5$								
Value	1.66	0.17	0.83	0.08	0.01	0.03	0.10	0.05
$\sigma = 1.0$								
Value	0.83	0.17	0.00	0.11	0.01	0.00	0.10	0.05
$\sigma = 2.0$								
Value	0.42	0.17	-0.42	0.18	0.02	-0.05	0.10	0.05

According to these parameter values for equation (2.1) when $\sigma = 0.5$, a 100 basis point increase in the ex ante real interest rates decreases the output gap by 166 basis points, a 100 basis point increase in government consumption in excess of its steady state value increases the output gap by 17 basis points, a 1 per cent taste (preference) shock increases the output gap by 83 basis points assuming the expected values for government consumption in excess of its steady state value and taste shock next period are unchanged at its zero value. For equation (2.3) the parameter values indicate that 100 basis point increase in the output gap increases inflation by 8 basis points and that a 100 basis point increase in the tax rate increases inflation by 5 basis

points. The other parameter values for equation (2.3) must be interpreted with caution since they assume fixed output gap.

When the value of σ is changed to 1 effects of the taste shock vanish (the values of κ_3 and γ_3 become zero) in addition to that the effect of real interest rates on demand decreases considerably (the value of γ_1 decreases from 1.66 to 0.83). However, when we change the value of σ to 2 the values of κ_3 and γ_3 become negative, which makes this assumption uncredible (since a positive demand shock should lead to increased demand and increased inflation). As can be seen below, the effects of different fiscal policies are the same in all three settings.

Using the parameter values from tables 2 and 3, the values of the b 's, c 's and d 's in (2.11) – (2.13) and (2.18) – (2.24) are given in table 4 on next page. In the first column, there are parameter values for the general setting in equations (2.11) – (2.13). In the next two columns, there are the parameter values for settings 1 and 2. In setting 1 $\hat{g}_t = \hat{\tau}_t = 0$ and therefore only the parameters of the last two terms in each of equations (2.11) – (2.13) need to be calculated. In setting 2 we have that $\hat{\tau}_t = 0$ and only the parameters of terms 1, 3 and 4 of each of the equations need to be calculated. In setting 3 $\hat{\tau}_t = 0$ still applies but \hat{g}_t is endogenous and in setting 4 both \hat{g}_t and $\hat{\tau}_t$ are endogenous. Therefore only the parameters of the last two terms in each of equation (2.11) – (2.13) need to be calculated for setting 4.

Table 4. The parameter values

Parameters	$\sigma = 0.5$								$\sigma = 1.0$								$\sigma = 2.0$							
	Settings								Settings								Settings							
	General	1	2	3: $\phi=2$	3: $\phi=-2$	4: $\phi=2$ $\theta=2$	4: $\phi=-2$ $\theta=2$	General	1	2	3: $\phi=2$	3: $\phi=-2$	4: $\phi=2$ $\theta=2$	4: $\phi=-2$ $\theta=2$	General	1	2	3: $\phi=2$	3: $\phi=-2$	4: $\phi=2$ $\theta=2$	4: $\phi=-2$ $\theta=2$			
b_1	0.09	-	0.09	-	-	-	-	0.12	-	0.12	-	-	-	-	0.14	-	0.14	-	-	-	-			
b_2	-0.06	-	-	-	-	-	-	-0.04	-	-	-	-	-	-0.02	-	-	-	-	-	-				
b_3	0.44	0.44	0.44	0.54	0.37	0.46	0.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.34	-0.34	-0.34	-0.47	-0.27	-0.44	-0.26			
b_4	0.13	0.13	0.13	0.15	0.11	0.13	0.10	0.08	0.08	0.08	0.11	0.07	0.10	0.06	0.05	0.05	0.05	0.07	0.04	0.06	0.04			
c_1	0.00	-	0.00	-	-	-	-	0.00	-	0.00	-	-	-	-	0.00	-	0.00	-	-	-	-			
c_2	0.05	-	-	-	-	-	-	0.05	-	-	-	-	-	-	0.05	-	-	-	-	-	-			
c_3	0.01	0.01	0.01	0.01	0.01	0.06	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.05	-0.03			
c_4	-0.09	-0.09	-0.09	-0.09	-0.09	-0.08	-0.08	-0.09	-0.09	-0.09	-0.09	-0.09	-0.09	-0.09	-0.10	-0.10	-0.10	-0.09	-0.10	-0.09	-0.09			
d_1	0.05	-	0.05	-	-	-	-	0.06	-	0.06	-	-	-	-	0.07	-	0.07	-	-	-	-			
d_2	0.04	-	-	-	-	-	-	0.05	-	-	-	-	-	-	0.06	-	-	-	-	-	-			
d_3	0.24	0.24	0.24	0.29	0.20	0.32	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.18	-0.18	-0.18	-0.25	-0.14	-0.30	-0.17			
d_4	-0.08	-0.08	-0.08	-0.06	-0.09	-0.05	-0.08	-0.10	-0.10	-0.10	-0.09	-0.11	-0.08	-0.10	-0.12	-0.12	-0.12	-0.11	-0.12	-0.10	-0.12			

3.2. Results

Looking at the parameter values for the general setting in table 3.3 when $\sigma = 0.5$ it shows that an increase in government consumption results in an increase in the output gap (since $b_1 > 0$), has negligible effects on inflation (since $c_1 = 0$) and results in higher interest rates (since $d_1 > 0$). A higher tax rate results in a smaller output gap ($b_2 < 0$), a higher inflation ($c_2 > 0$) and results in higher interest rates ($d_2 > 0$). A positive taste shock results in an increase in the output gap ($b_3 > 0$), an increase in inflation ($c_3 > 0$) and an increase in the interest rate ($d_3 > 0$). Finally, a positive productivity shock results in an increase in the output gap ($b_4 > 0$), decreases in inflation ($c_4 < 0$) and decreases in the interest rate ($d_4 < 0$).

From setting 1 it can be seen that by setting government consumption and the tax rate equal to their steady state values decreases fluctuation in output gap, inflation and interest rates as compared to the general setting.²⁶ This is also the case for setting 2 in which only the tax rate is set equal to its steady state value. However, setting both government consumption and the tax rate equal to their steady state values results in less fluctuation in the three endogenous variables than if only the tax rate is fixed.

In settings 3 the tax rate is set equal to its steady state value but deviations in government consumption from its steady state moves with the output gap. First, a positive correlation is assumed between the two variables ($\phi > 0$). In this case, fluctuation in the output gap is bigger than in setting 1 but it is uncertain if it is bigger than in the general setting or in setting 2. Fluctuation in inflation seem to be equal to what it is in setting 1 and 2 but smaller than in the general setting. It is uncertain if fluctuation in interest rates is bigger than in the general setting, setting 1 or setting 2.

Second, a negative correlation is assumed between the two variables ($\phi < 0$) in setting 3. In this case, fluctuation in the output gap is less than in then general setting, setting 1 and in setting 2. Fluctuation in inflation seem to be equal to what it is in setting 1 and 2 but smaller than in the general setting. It is uncertain if there is more

²⁶ Remember that if $y = \sum_{i=1}^n a_i x_i$ and the x 's are independent of each other, then its variance (a

measure of fluctuations) can be written as: $Var(y) = \sum_{i=1}^n a_i^2 Var(x_i)$.

fluctuation in the interest rate than in the general setting, setting 1 or setting 2. Setting 3 with negative correlation between the two variables results in less fluctuation in output gap than setting 3 with positive correlation.

In setting 4 progressive taxation is added to setting 3. In this case the tax rate has positive correlation with output gap ($\theta > 0$). Compared to setting 3 this setting results in less fluctuation in the output gap. It is, however, uncertain what effects this has on fluctuations in inflation and interest rates as compared to setting 3. Compared to the general setting and settings 1 and 2 this results in less fluctuation in the output gap if correlation between deviations in government consumption from its steady state level and the output gap is negative. If the correlation is positive, setting 4 results in more fluctuations in the output gap compared to setting 1 while the comparison with the general setting and setting 2 does not give a clear cut answer. Comparison for the inflation and interest rates between setting 4 on one hand, and the general setting, setting 1 and setting 2 on the other, does not give any clear cut answers.

The conclusions drawn for the case when $\sigma = 0.5$ also hold when $\sigma = 1$ and $\sigma = 2$ according to table 4.

4. Conclusions

If the objective of fiscal policy is to minimize fluctuations, the results in this paper can be used to draw some conclusions concerning fiscal policy in this model economy. The best fiscal policy in this respect is to use progressive taxation with counter-cyclical government consumption since this results in the smallest fluctuation in the output gap (setting 4: $\phi = -2$, $\theta = 2$). The second-best fiscal policy is to use counter-cyclical government consumption with fixed tax rates equal to its steady state value (setting 3: $\phi = -2$), and the third-best is to fix both government consumption and the tax rate equal to their steady state values (setting 1). The fourth-best fiscal policy is to fix the tax rate equal to its steady state value and allow deviation in government consumption from its steady state value to fluctuate (setting 2). Finally, the worst fiscal policy is to allow deviation in both government consumption and the tax rate from their steady state values to fluctuate around their steady state values (general setting).

However, conducting a counter-cyclical fiscal policy using government consumption requires a lot of information about where in the business cycle we are at

any given time. Also, it may take time to put decisions into action, and the policy could therefore increase fluctuation instead of mitigating it. As was discussed in the introduction to the paper, this may explain why many of the OECD countries have been unsuccessful in implementing an effective counter-cyclical fiscal policy with respect to government consumption in recent decades, given that they were trying to follow such a fiscal policy. As the results in this paper indicate, an unsuccessful fiscal policy (setting 4: $\phi=2$, $\theta=2$ and setting 3: $\phi=2$) results in more fluctuation in the output gap than simply allowing government consumption to equal its steady state value. Hence, for many of the OECD countries, allowing government consumption to grow at a predetermined rate would have resulted in less fluctuation in the output gap and, hence, would have resulted in less fluctuation in output.

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Mathematical Appendix

A.1. Equation (1.4).

The Lagrangian for the problem is the following:

$$L = \int_{j=0}^1 p_{jt} c_{jt} dj + \lambda_t \left\{ C_t - \left[\int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \right\}$$

where λ is the Lagrange multiplier. The first order conditions are the following in addition to the constraint in (1.2):

$$L_{c_{jt}} = p_{jt} - \lambda_t \left[\int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0 \quad \forall j$$

or

$$\begin{aligned} p_{jt} - \lambda_t \left[\int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0 &\Rightarrow c_{jt}^{-\frac{1}{\theta}} = \left(\frac{p_{jt}}{\lambda_t} \right) \left[\int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{1-\theta}} \\ \Rightarrow c_{jt} = \left(\frac{p_{jt}}{\lambda_t} \right)^{-\theta} \left[\int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} &\Rightarrow c_{jt} = \left(\frac{p_{jt}}{\lambda_t} \right)^{-\theta} C_t \end{aligned} \tag{A.1}$$

Plugging this into the constraint in (1.2) gives and solving for λ gives the aggregate price level (P):

$$C_t = \left[\int_{j=0}^1 \left\{ \left(\frac{P_{jt}}{\lambda_t} \right)^{-\theta} C_t \right\}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \lambda_t^\theta \left[\int_{j=0}^1 P_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t$$

$$\Rightarrow \lambda_t = \left[\int_{j=0}^1 P_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \equiv P_t$$

Plugging this result into (A.1) gives equation (1.4):

$$c_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} C_t$$

A.2. Equations (1.6) – (1.8)

The Lagrangian for the problem is the following:

$$L = E_t \sum_{i=0}^{\infty} \beta^i \left[\begin{aligned} & \left(\frac{\psi_{t+i} C_{t+i}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right. \\ & \left. + \lambda_{t+i} \left\{ \begin{aligned} & \left(\frac{W_{t+i}}{P_{t+i}} \right) (1-\tau_{t+i}) N_{t+i} + \frac{M_{t+i-1}}{P_{t+i}} \right. \right. \\ & \left. \left. + (1+i_{t+i-1}) \left(\frac{B_{t+i-1}}{P_{t+i}} \right) + \Pi_{t+i} - C_{t+i} - \frac{M_{t+i}}{P_{t+i}} - \frac{B_{t+i}}{P_{t+i}} \right\} \right. \end{aligned} \right]$$

The first order conditions are the following in addition to the constraint in (1.5):

$$L_{C_t} = \psi_t^{1-\sigma} C_t^{-\sigma} - \lambda_t = 0 \quad (\text{A.2})$$

$$L_{N_t} = -\chi N_t^\eta + \lambda_t \left(\frac{W_t}{P_t} \right) (1-\tau_t) = 0 \quad (\text{A.3})$$

$$L_{M_t} = \gamma \left(\frac{M_t}{P_t} \right)^{-b} \frac{1}{P_t} - \frac{\lambda_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{P_{t+1}} = 0 \quad (\text{A.4})$$

$$L_{B_t} = -\frac{\lambda_t}{P_t} + \beta(1+i_t) E_t \frac{\lambda_{t+1}}{P_{t+1}} = 0 \quad (\text{A.5})$$

Eliminating λ from (A.2) and plugging that into (A.3) gives (1.6):

$$-\chi N_t^\eta + \psi_t^{1-\sigma} C_t^{-\sigma} \left(\frac{W_t}{P_t} \right) (1-\tau_t) = 0 \Rightarrow \left(\frac{W_t}{P_t} \right) (1-\tau_t) = \frac{\chi N_t^\eta}{\psi_t^{1-\sigma} C_t^{-\sigma}}$$

Eliminating λ from (A.2) and plugging that into (A.5) gives (1.7):

$$\begin{aligned}
& -\frac{\psi_t^{1-\sigma} C_t^{-\sigma}}{P_t} + \beta(1+i_t)E_t \frac{\psi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma}}{P_{t+1}} = 0 \\
\Rightarrow & \frac{\psi_t^{1-\sigma} C_t^{-\sigma}}{P_t} = \beta(1+i_t)E_t \frac{\psi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma}}{P_{t+1}}
\end{aligned}$$

Eliminating $\beta E_t \frac{\lambda_{t+1}}{P_{t+1}}$ from (A.4), plugging that into (A.5), eliminating λ from (A.2)

and plugging that into the resulting equation gives (1.8):

$$\begin{aligned}
& -\frac{\psi_t^{1-\sigma} C_t^{-\sigma}}{P_t} + (1+i_t) \left\{ -\gamma \left(\frac{M_t}{P_t} \right)^{-b} \frac{1}{P_t} + \frac{\psi_t^{1-\sigma} C_t^{-\sigma}}{P_t} \right\} = 0 \\
\Rightarrow & i_t \psi_t^{1-\sigma} C_t^{-\sigma} - \gamma(1+i_t) \left(\frac{M_t}{P_t} \right)^{-b} = 0 \\
\Rightarrow & \frac{\gamma \left(\frac{M_t}{P_t} \right)^{-b}}{\psi_t^{1-\sigma} C_t^{-\sigma}} = \frac{i_t}{1+i_t}
\end{aligned}$$

A.3. Equation (1.11)

Plugging equation (1.10) into the composite production definition $Y_t = \left[\int_{j=0}^1 y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$

gives:

$$Y_t = \left[\int_{j=0}^1 \left\{ (1+\alpha_t) c_{jt} \right\}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \left[(1+\alpha_t)^{\frac{\theta-1}{\theta}} \int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = (1+\alpha_t) \left[\int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

Since the term in the brackets is composite consumption: $C_t = \left[\int_{j=0}^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$, this

gives the first part of equation (1.11). The second part comes from the definition of total demand.

A.4. Equation (1.14)

The Lagrangian for the problem is the following:

$$L = \left(\frac{W_t}{P_t} \right) N_{jt} + \lambda_t (y_{jt} - Z_t N_{jt})$$

The first order condition is the following in addition to the constraint in (1.12):

$$L_{N_j} = \frac{W_t}{P_t} - \lambda_t Z_t = 0$$

This condition gives equation (1.14):

$$\lambda_t = \frac{W_t / P_t}{Z_t} \equiv mc_t$$

A.5. Equation (1.16)

Substituting (1.10) into (1.15) gives:

$$E_t \sum_{i=0}^{\infty} \omega^i \tilde{\beta}_{t,t+i} \left[\left(\frac{p_{j,t}}{P_{t+i}} \right) c_{j,t+i} - mc_{t+i} c_{j,t+i} \right] (1 + a_{t+i}) \quad (\text{A.6})$$

Substituting (1.4) into (A.6) gives:

$$\begin{aligned} E_t \sum_{i=0}^{\infty} \omega^i \tilde{\beta}_{t,t+i} \left[\left(\frac{p_{j,t}}{P_{t+i}} \right) \left(\frac{p_{j,t}}{P_{t+i}} \right)^{-\theta} C_{t+i} - mc_{t+i} \left(\frac{p_{j,t}}{P_{t+i}} \right)^{-\theta} C_{t+i} \right] (1 + a_{t+i}) \\ = E_t \sum_{i=0}^{\infty} \omega^i \tilde{\beta}_{t,t+i} \left[\left(\frac{p_{j,t}}{P_{t+i}} \right)^{1-\theta} - mc_{t+i} \left(\frac{p_{j,t}}{P_{t+i}} \right)^{-\theta} \right] Y_{t+i} \end{aligned} \quad (\text{A.7})$$

Differentiating (A.7) with respect to p_j , setting $p_j = p^*$ (optimal price) and rearranging gives:

$$\begin{aligned} E_t \sum_{i=0}^{\infty} \omega^i \tilde{\beta}_{t,t+i} \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} \frac{1}{P_{t+i}} + \theta mc_{t+i} \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta-1} \frac{1}{P_{t+i}} \right] Y_{t+i} = 0 \\ \Rightarrow E_t \sum_{i=0}^{\infty} \omega^i \tilde{\beta}_{t,t+i} \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right) + \theta mc_{t+i} \right] \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} Y_{t+i} = 0 \end{aligned} \quad (\text{A.8})$$

Substituting equation (1.11) into $\tilde{\beta}_{t,t+i} = \beta^i \frac{\psi_{t+i}^{1-\sigma} C_{t+i}^{-\sigma}}{\psi_t^{1-\sigma} C_t^{-\sigma}}$ and the results from that into

(A.8) gives equation (1.16):

$$\begin{aligned} E_t \sum_{i=0}^{\infty} \omega^i \beta^i \frac{\psi_{t+i}^{1-\sigma} Y_{t+i}^{-\sigma} (1+a_t)^{-\sigma}}{\psi_t^{1-\sigma} Y_t^{-\sigma} (1+a_{t+i})^{-\sigma}} \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right) + \theta mc_{t+i} \right] \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} Y_{t+i} = 0 \\ \Rightarrow E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right) + \theta mc_{t+i} \right] \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^{\sigma} = 0 \end{aligned}$$

A.6. Equation (2.1)

Linearizing equation (1.7) around the zero inflation steady state gives:²⁷

$$\begin{aligned}
\frac{(1+\hat{\psi}_t)^{1-\sigma}(1+\hat{c}_t)^{-\sigma}(C^{SS})^{-\sigma}}{(1+\hat{p}_t)P^{SS}} &= (1+\hat{i}_t)\beta^{-1}E_t\left(\frac{(1+\hat{\psi}_{t+1})^{1-\sigma}(1+\hat{c}_{t+1})^{-\sigma}(C^{SS})^{-\sigma}}{(1+\hat{p}_{t+1})P^{SS}}\right) \\
\Rightarrow \frac{(1+\hat{\psi}_t)^{1-\sigma}(1+\hat{c}_t)^{-\sigma}}{(1+\hat{p}_t)} &= (1+\hat{i}_t)E_t\left(\frac{(1+\hat{\psi}_{t+1})^{1-\sigma}(1+\hat{c}_{t+1})^{-\sigma}}{(1+\hat{p}_{t+1})}\right) \\
\Rightarrow (1+\hat{\psi}_t)^{1-\sigma}(1+\hat{c}_t)^{-\sigma}(1+\hat{p}_t)^{-1} &= (1+\hat{i}_t)E_t\left[(1+\hat{\psi}_{t+1})^{1-\sigma}(1+\hat{c}_{t+1})^{-\sigma}(1+\hat{p}_{t+1})^{-1}\right] \\
\Rightarrow [1+(1-\sigma)\hat{\psi}_t][1-\sigma\hat{c}_t](1-\hat{p}_t) &= (1+\hat{i}_t)E_t\{[1+(1-\sigma)\hat{\psi}_{t+1}][1-\sigma\hat{c}_{t+1}](1-\hat{p}_{t+1})\} \\
\Rightarrow [1+(1-\sigma)\hat{\psi}_t-\sigma\hat{c}_t-\hat{p}_t] &= (1+\hat{i}_t)E_t[1+(1-\sigma)\hat{\psi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{p}_{t+1}] \\
\Rightarrow [1+(1-\sigma)\hat{\psi}_t-\sigma\hat{c}_t-\hat{p}_t] &= (1+\hat{i}_t)[1+(1-\sigma)E_t\hat{\psi}_{t+1}-\sigma E_t\hat{c}_{t+1}-E_t\hat{p}_{t+1}] \\
\Rightarrow 1+(1-\sigma)\hat{\psi}_t-\sigma\hat{c}_t-\hat{p}_t &= 1+\hat{i}_t+(1-\sigma)E_t\hat{\psi}_{t+1}-\sigma E_t\hat{c}_{t+1}-E_t\hat{p}_{t+1} \\
\Rightarrow (1-\sigma)\hat{\psi}_t-\sigma\hat{c}_t-\hat{p}_t &= \hat{i}_t+(1-\sigma)E_t\hat{\psi}_{t+1}-\sigma E_t\hat{c}_{t+1}-E_t\hat{p}_{t+1} \tag{A.9}
\end{aligned}$$

Solving equation (A.9) for \hat{c}_t gives:

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \frac{1-\sigma}{\sigma}(E_t\hat{\psi}_{t+1} - \hat{\psi}_t) \tag{A.10}$$

where $E_t\pi_{t+1} = E_t\hat{p}_{t+1} - \hat{p}_t$ is the expected inflation rate. Using the fact that

$$Y_t = C_t + G_t \Rightarrow (1+\hat{y}_t)Y^{SS} = (1+\hat{c}_t)C^{SS} + (1+\hat{g}_t)G^{SS} \Rightarrow \hat{c}_t = \hat{y}_t \frac{Y^{SS}}{C^{SS}} - \hat{g}_t \frac{G^{SS}}{C^{SS}} \quad \text{and}$$

plugging it into equation (A.10) gives equation (2.1):

$$\begin{aligned}
\hat{y}_t \frac{Y^{SS}}{C^{SS}} - \hat{g}_t \frac{G^{SS}}{C^{SS}} &= E_t\left(\hat{y}_{t+1} \frac{Y^{SS}}{C^{SS}} - \hat{g}_{t+1} \frac{G^{SS}}{C^{SS}}\right) - \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \frac{1-\sigma}{\sigma}(E_t\hat{\psi}_{t+1} - \hat{\psi}_t) \\
\Rightarrow \hat{y}_t \frac{Y^{SS}}{C^{SS}} &= -E_t\hat{g}_{t+1} \frac{G^{SS}}{C^{SS}} + E_t\hat{y}_{t+1} \frac{Y^{SS}}{C^{SS}} - \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) + \hat{g}_t \frac{G^{SS}}{C^{SS}} - \frac{1-\sigma}{\sigma}(E_t\hat{\psi}_{t+1} - \hat{\psi}_t) \\
\Rightarrow \hat{y}_t &= E_t\hat{y}_{t+1} - \frac{1}{\sigma} \frac{C^{SS}}{Y^{SS}}(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \frac{G^{SS}}{Y^{SS}}(E_t\hat{g}_{t+1} - \hat{g}_t) - \frac{1-\sigma}{\sigma} \frac{C^{SS}}{Y^{SS}}(E_t\hat{\psi}_{t+1} - \hat{\psi}_t) \\
\Rightarrow \hat{y}_t &= E_t\hat{y}_{t+1} - \gamma_1(\hat{i}_t - E_t\hat{\pi}_{t+1}) - \gamma_2(E_t\hat{g}_{t+1} - \hat{g}_t) - \gamma_3(E_t\hat{\psi}_{t+1} - \hat{\psi}_t)
\end{aligned}$$

A.7. Equation (2.2)

The first order condition in equation (1.16) can be written in the following way:

$$\begin{aligned}
E_t \sum_{i=0}^{\infty} \omega^i \beta^i (1-\theta) &\left(\frac{1}{P_t^*}\right) \left(\frac{P_t^*}{P_{t+i}}\right)^{1-\theta} \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^\sigma \\
+ E_t \sum_{i=0}^{\infty} \omega^i \beta^i \theta mc_{t+i} &\left(\frac{1}{P_t^*}\right) \left(\frac{P_t^*}{P_{t+i}}\right)^{-\theta} \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^\sigma = 0
\end{aligned}$$

²⁷ For reviewing the rules of linear approximation see Walsh, C. E. (2003), chapter 2.7.4.

$$\begin{aligned} &\Rightarrow (1-\theta) \left(\frac{P_t^*}{P_t} \right) E_t \sum_{i=0}^{\infty} \omega^i \beta^i \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^\sigma \left(\frac{P_t}{P_{t+i}} \right)^{1-\theta} \\ &+ \theta E_t \sum_{i=0}^{\infty} \omega^i \beta^i m c_{t+i} \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^\sigma \left(\frac{P_t}{P_{t+i}} \right)^{-\theta} = 0 \end{aligned}$$

Setting $Q = p^*/P$ and rearranging gives:²⁸

$$\begin{aligned} &Q_t E_t \sum_{i=0}^{\infty} \omega^i \beta^i \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^\sigma \left(\frac{P_t}{P_{t+i}} \right)^{1-\theta} \\ &= \frac{\theta}{\theta-1} E_t \sum_{i=0}^{\infty} \omega^i \beta^i m c_{t+i} \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^\sigma \left(\frac{P_t}{P_{t+i}} \right)^{-\theta} \end{aligned} \quad (\text{A.10})$$

Linearizing (A.10) around the zero inflation steady state gives:²⁹

$$\begin{aligned} &(1+\hat{q}_t)(Y^{SS})^{1-\sigma} [(1+a)^{SS}]^\sigma \times \\ &E_t \sum_{i=0}^{\infty} \omega^i \beta^i (1+\hat{\psi}_{t+i})^{1-\sigma} (1+\hat{y}_{t+i})^{1-\sigma} (1+\hat{a}_{t+i})^\sigma (1+\hat{p}_t)^{1-\theta} (1+\hat{p}_{t+i})^{\theta-1} \\ &= \frac{\theta}{\theta-1} (Y^{SS})^{1-\sigma} [(1+a)^{SS}]^\sigma m c^{SS} \times \\ &E_t \sum_{i=0}^{\infty} \omega^i \beta^i (1+\hat{m}c_{t+i})(1+\hat{\psi}_{t+i})^{1-\sigma} (1+\hat{y}_{t+i})^{1-\sigma} (1+\hat{a}_{t+i})^\sigma (1+\hat{p}_t)^{-\theta} (1+\hat{p}_{t+i})^\theta \\ &\Rightarrow (1+\hat{q}_t) E_t \sum_{i=0}^{\infty} \omega^i \beta^i (1+\hat{\psi}_{t+i})^{1-\sigma} (1+\hat{y}_{t+i})^{1-\sigma} (1+\hat{a}_{t+i})^\sigma (1+\hat{p}_t)(1+\hat{p}_{t+i})^{\theta-1} \\ &= \frac{\theta}{\theta-1} m c^{SS} E_t \sum_{i=0}^{\infty} \omega^i \beta^i (1+\hat{m}c_{t+i})(1+\hat{\psi}_{t+i})^{1-\sigma} (1+\hat{y}_{t+i})^{1-\sigma} (1+\hat{a}_{t+i})^\sigma (1+\hat{p}_{t+i})^\theta \\ &\Rightarrow (1+\hat{q}_t) E_t \sum_{i=0}^{\infty} \omega^i \beta^i [1+(1-\sigma)\hat{\psi}_{t+i}] [1+(1-\sigma)\hat{y}_{t+i}] [1+\sigma\hat{a}_{t+i}] (1+\hat{p}_t) [1+(\theta-1)\hat{p}_{t+i}] \\ &= \frac{\theta}{\theta-1} m c^{SS} E_t \sum_{i=0}^{\infty} \omega^i \beta^i (1+\hat{m}c_{t+i}) [1+(1-\sigma)\hat{\psi}_{t+i}] [1+(1-\sigma)\hat{y}_{t+i}] [1+\sigma\hat{a}_{t+i}] (1+\hat{\theta}_{t+i}) \\ &\Rightarrow (1+\hat{q}_t) E_t \sum_{i=0}^{\infty} \omega^i \beta^i [1+(1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \hat{p}_t + (\theta-1)\hat{p}_{t+i}] \\ &= \frac{\theta}{\theta-1} m c^{SS} E_t \sum_{i=0}^{\infty} \omega^i \beta^i [1+\hat{m}c_{t+i} + (1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \hat{\theta}_{t+i}] \end{aligned} \quad (\text{A.11})$$

Equation (A.11) is approximated by:

²⁸ Note that $p^* = P$ in steady state and, hence, $Q = 1$ in it.

²⁹ For reviewing the rules of linear approximation see Walsh, C. E. (2003), chapter 2.7.4.

$$\begin{aligned}
& \frac{1}{1-\omega\beta} + \frac{\hat{q}_t}{1-\omega\beta} + E_t \sum_{i=0}^{\infty} \omega^i \beta^i [(1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \hat{p}_t + (\theta-1)\hat{p}_{t+i}] \\
&= \frac{1}{1-\omega\beta} \frac{\theta}{\theta-1} mc^{SS} + \frac{\theta}{\theta-1} mc^{SS} E_t \sum_{i=0}^{\infty} \omega^i \beta^i [\hat{m}c_{t+i} + (1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \theta\hat{p}_{t+i}]
\end{aligned} \tag{A.12}$$

In appendix (A.8) we show that the marginal cost in steady state is:

$$mc^{SS} = \frac{\theta-1}{\theta} \tag{A.13}$$

Plugging this into (A.12) gives:

$$\begin{aligned}
& \Rightarrow \frac{1}{1-\omega\beta} + \frac{\hat{q}_t}{1-\omega\beta} + E_t \sum_{i=0}^{\infty} \omega^i \beta^i [(1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \hat{p}_t + (\theta-1)\hat{p}_{t+i}] \\
&= \frac{1}{1-\omega\beta} + E_t \sum_{i=0}^{\infty} \omega^i \beta^i [\hat{m}c_{t+i} + (1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \theta\hat{p}_{t+i}] \\
& \Rightarrow \frac{\hat{q}_t}{1-\omega\beta} + E_t \sum_{i=0}^{\infty} \omega^i \beta^i [(1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \hat{p}_t + (\theta-1)\hat{p}_{t+i}] \\
&= E_t \sum_{i=0}^{\infty} \omega^i \beta^i [\hat{m}c_{t+i} + (1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \theta\hat{p}_{t+i}] \\
& \Rightarrow \frac{\hat{q}_t}{1-\omega\beta} + \frac{\hat{p}_t}{1-\omega\beta} + E_t \sum_{i=0}^{\infty} \omega^i \beta^i [(1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + (\theta-1)\hat{p}_{t+i}] \\
&= E_t \sum_{i=0}^{\infty} \omega^i \beta^i [\hat{m}c_{t+i} + (1-\sigma)\hat{\psi}_{t+i} + (1-\sigma)\hat{y}_{t+i} + \sigma\hat{a}_{t+i} + \theta\hat{p}_{t+i}] \\
& \Rightarrow \frac{\hat{q}_t}{1-\omega\beta} + \frac{\hat{p}_t}{1-\omega\beta} + E_t \sum_{i=0}^{\infty} \omega^i \beta^i [(\theta-1)\hat{p}_{t+i}] \\
&= E_t \sum_{i=0}^{\infty} \omega^i \beta^i [\hat{m}c_{t+i} + \theta\hat{p}_{t+i}] \\
& \Rightarrow \frac{\hat{q}_t}{1-\omega\beta} + \frac{\hat{p}_t}{1-\omega\beta} = E_t \sum_{i=0}^{\infty} \omega^i \beta^i [\hat{m}c_{t+i} + \hat{p}_{t+i}] \\
& \Rightarrow \hat{q}_t + \hat{p}_t = (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i [E_t \hat{m}c_{t+i} + E_t \hat{p}_{t+i}]
\end{aligned} \tag{A.14}$$

(A.14) can be rewritten as:

$$\begin{aligned}
& \hat{q}_t + \hat{p}_t = (1-\omega\beta)(\hat{m}c_t + \hat{p}_t) + \omega\beta[E_t \hat{q}_{t+1} + E_t \hat{p}_{t+1}] \\
& \Rightarrow \hat{q}_t = (1-\omega\beta)\hat{m}c_t + \omega\beta[E_t \hat{q}_{t+1} + E_t \hat{p}_{t+1}]
\end{aligned} \tag{A.15}$$

In appendix A.9 we show that \hat{q} can be written as:

$$\hat{q}_t = \frac{\omega}{1-\omega} \hat{\pi}_t \tag{A.16}$$

Plugging (A.16) into (A.15) gives equation (2.2):

$$\begin{aligned}
\frac{\omega}{1-\omega} \hat{\pi}_t &= (1-\omega\beta) \hat{m}c_t + \omega\beta \left[\frac{\omega}{1-\omega} E_t \hat{\pi}_{t+1} + E_t \hat{\pi}_{t+1} \right] \\
\Rightarrow \frac{\omega}{1-\omega} \hat{\pi}_t &= (1-\omega\beta) \hat{m}c_t + \omega\beta \frac{1}{1-\omega} E_t \hat{\pi}_{t+1} \\
\Rightarrow \hat{\pi}_t &= \frac{(1-\omega\beta)(1-\omega)}{\omega} \hat{m}c_t + \beta E_t \hat{\pi}_{t+1} \\
\Rightarrow \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa \hat{m}c_t
\end{aligned}$$

A.8. Equation (A.13)

In steady state, prices are perfectly flexible, i.e. $\omega = 0$. Setting $\omega = 0$ in equation (1.16), noting that $p^* = P$ when prices are flexible, gives the following:

$$\begin{aligned}
E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left[(1-\theta) \left(\frac{P_t}{P_{t+i}} \right) + \theta mc_{t+i} \right] \left(\frac{1}{P_t} \right) \left(\frac{P_t}{P_{t+i}} \right)^{-\theta} \psi_{t+i}^{1-\sigma} Y_{t+i}^{1-\sigma} (1+a_{t+i})^\sigma &= 0 \\
\Rightarrow \beta^0 \left[(1-\theta) \left(\frac{P_t}{P_t} \right) + \theta mc_{t+i} \right] \left(\frac{1}{P_t} \right) \left(\frac{P_t}{P_t} \right)^{-\theta} \psi_t^{1-\sigma} Y_t^{1-\sigma} (1+a_t)^\sigma &= 0 \\
\Rightarrow \left[(1-\theta) + \theta mc_{t+i} \right] \left(\frac{1}{P_t} \right) \psi_t^{1-\sigma} Y_t^{1-\sigma} (1+a_t)^\sigma &= 0 \\
\Rightarrow (1-\theta) + \theta mc_{t+i} &= 0 \\
\Rightarrow mc^{ss} &= \frac{\theta-1}{\theta}
\end{aligned}$$

A.9. Equation (A.16)

Rearranging equation (1.17) gives:

$$\begin{aligned}
P_t^{1-\theta} &= (1-\omega) p_t^{*1-\theta} + \omega P_{t-1}^{1-\theta} \\
\Rightarrow 1 &= (1-\omega) \left(\frac{P_t^*}{P_t} \right)^{1-\theta} + \omega \left(\frac{P_{t-1}}{P_t} \right)^{1-\theta} \\
\Rightarrow 1 &= (1-\omega) q_t^{1-\theta} + \omega \left(\frac{1}{1+\pi_t} \right)^{1-\theta} \\
\Rightarrow 1 &= (1-\omega) q_t^{1-\theta} + \omega (1+\pi_t)^{\theta-1}
\end{aligned}$$

Linearizing this equation around the zero inflation steady state inflation rate gives:

$$\begin{aligned}
1 &= (1-\omega)(1+\hat{q}_t)^{1-\theta} + \omega(1+\hat{\pi}_t)^{\theta-1} \\
\Rightarrow 1 &= (1-\omega)[1+(1-\theta)\hat{q}_t] + \omega[1+(\theta-1)\hat{\pi}_t] \\
\Rightarrow 0 &= (1-\omega)(1-\theta)\hat{q}_t - \omega(1-\theta)\hat{\pi}_t \\
\Rightarrow \omega\hat{\pi}_t &= (1-\omega)\hat{q}_t \\
\Rightarrow \hat{q}_t &= \frac{\omega}{1-\omega}\hat{\pi}_t
\end{aligned}$$

A.10. Equation (2.3)

Rearranging equation (1.14) and linearizing it around the zero inflation steady state gives:³⁰

$$\begin{aligned}
(1+\hat{m}c_t)(1+\hat{z}_t)(1+\hat{p}_t)mc^{SS} &= (1+\hat{w}_t)W^{SS} \\
\Rightarrow (1+\hat{m}c_t)(1+\hat{z}_t)(1+\hat{p}_t) &= (1+\hat{w}_t) \\
\Rightarrow 1+\hat{m}c_t + \hat{z}_t + \hat{p}_t &= 1+\hat{w}_t \\
\Rightarrow \hat{m}c_t + \hat{z}_t + \hat{p}_t &= \hat{w}_t \\
\Rightarrow \hat{m}c_t &= \hat{w}_t - \hat{p}_t - \hat{z}_t
\end{aligned} \tag{A.17}$$

Using (1.12) and $Y_t = \left[\int_{j=0}^1 y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$ we can write total production in the economy as

$$Y_t = \left[\int_{j=0}^1 Z_t N_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \left[Z_t^{\frac{\theta-1}{\theta}} \int_{j=0}^1 N_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = Z_t \left[\int_{j=0}^1 N_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = Z_t N_t \tag{A.18}$$

where N is the composite labor use. Linearizing equation (A.18) around the zero inflation steady state gives:³¹

$$\begin{aligned}
(1+\hat{y}_t)Y^{SS} &= (1+\hat{z}_t)(1+\hat{n}_t)N^{SS} \\
\Rightarrow (1+\hat{y}_t) &= (1+\hat{z}_t)(1+\hat{n}_t) \\
\Rightarrow 1+\hat{y}_t &= 1+\hat{z}_t + \hat{n}_t \\
\Rightarrow \hat{y}_t &= \hat{z}_t + \hat{n}_t \\
\Rightarrow \hat{z}_t &= \hat{y}_t - \hat{n}_t
\end{aligned} \tag{A.19}$$

Plugging (A.19) into (A.17) gives:

$$\Rightarrow \hat{m}c_t = \hat{w}_t - \hat{p}_t - (\hat{y}_t - \hat{n}_t) \tag{A.20}$$

Linearizing equation (1.6) around the zero inflation steady state gives:³²

³⁰ Remember that Z has an average of 1 and, hence, its steady state value is 1, i.e. $Z^{SS} = 1$. Also, from equation 1.14, $mc^{SS} = W^{SS}$ since this is a zero inflation steady state.

³¹ From equation (A.19) we have that $Y^{SS} = N^{SS}$.

$$\begin{aligned}
\frac{(1+\hat{w}_t)W^{SS}}{(1+\hat{p}_t)}(1-\hat{\tau}_t)(1-\tau^{SS}) &= \frac{\chi(1+\hat{n}_t)^\eta(N^{SS})^\eta}{(1+\hat{\psi}_t)^{1-\sigma}(1+\hat{c}_t)^{-\sigma}(C^{SS})^{-\sigma}} \\
\Rightarrow \frac{(1+\hat{w}_t)}{(1+\hat{p}_t)}(1-\hat{\tau}_t) &= \frac{(1+\hat{n}_t)^\eta}{(1+\hat{\psi}_t)^{1-\sigma}(1+\hat{c}_t)^{-\sigma}} \\
\Rightarrow (1+\hat{w}_t)(1-\hat{\tau}_t)(1+\hat{\psi}_t)^{1-\sigma}(1+\hat{c}_t)^{-\sigma} &= (1+\hat{n}_t)^\eta(1+\hat{p}_t) \\
\Rightarrow (1+\hat{w}_t-\hat{\tau}_t)[1+(1-\sigma)\hat{\psi}_t][1-\sigma\hat{c}_t] &= (1+\eta\hat{n}_t)(1+\hat{p}_t) \\
\Rightarrow 1+\hat{w}_t-\hat{\tau}_t+(1-\sigma)\hat{\psi}_t-\sigma\hat{c}_t &= 1+\eta\hat{n}_t+\hat{p}_t \\
\Rightarrow \hat{w}_t-\hat{p}_t &= \sigma\hat{c}_t+\eta\hat{n}_t-(1-\sigma)\hat{\psi}_t+\hat{\tau}_t
\end{aligned} \tag{A.21}$$

Plugging equation (A.21) into equation (A.20) gives

$$\hat{m}c_t = \sigma\hat{c}_t + \eta\hat{n}_t - (1-\sigma)\hat{\psi}_t - (\hat{y}_t - \hat{n}_t) + \hat{\tau}_t \tag{A.22}$$

Using the results in (A.19) gives:

$$\begin{aligned}
\hat{m}c_t &= \sigma\hat{c}_t + \eta(\hat{y}_t - \hat{z}_t) - (1-\sigma)\hat{\psi}_t - \hat{z}_t + \hat{\tau}_t \\
\Rightarrow \hat{m}c_t &= \sigma\hat{c}_t + \eta\hat{y}_t - (1-\sigma)\hat{\psi}_t - (1+\eta)\hat{z}_t + \hat{\tau}_t
\end{aligned} \tag{A.23}$$

Using the fact that $\hat{c}_t = \hat{y}_t \frac{Y^{CC}}{C^{SS}} - \hat{g}_t \frac{G^{SS}}{C^{SS}}$, (A.23) becomes:³³

$$\begin{aligned}
\hat{m}c_t &= \sigma \left(\hat{y}_t \frac{Y^{CC}}{C^{SS}} - \hat{g}_t \frac{G^{SS}}{C^{SS}} \right) + \eta\hat{y}_t - (1-\sigma)\hat{\psi}_t - (1+\eta)\hat{z}_t + \hat{\tau}_t \\
\Rightarrow \hat{m}c_t &= \frac{\sigma}{C^{SS}/Y^{SS}} \hat{y}_t - \sigma \frac{G^{SS}/Y^{SS}}{C^{SS}/Y^{SS}} \hat{g}_t + \eta\hat{y}_t - (1-\sigma)\hat{\psi}_t - (1+\eta)\hat{z}_t + \hat{\tau}_t \\
\Rightarrow \hat{m}c_t &= \frac{\sigma}{\alpha} \hat{y}_t - \sigma \frac{\delta}{\alpha} \hat{g}_t + \eta\hat{y}_t - (1-\sigma)\hat{\psi}_t - (1+\eta)\hat{z}_t + \hat{\tau}_t \\
\Rightarrow \hat{m}c_t &= \left(\eta + \frac{\sigma}{\alpha} \right) \hat{y}_t - \sigma \frac{\delta}{\alpha} \hat{g}_t - (1-\sigma)\hat{\psi}_t - (1+\eta)\hat{z}_t + \hat{\tau}_t
\end{aligned} \tag{A.24}$$

Plugging (A.24) into (2.2) then gives equation (2.3):

$$\begin{aligned}
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \frac{(1-\omega\beta)(1-\omega)}{\omega} \left[\left(\eta + \frac{\sigma}{\alpha} \right) \hat{y}_t - \sigma \frac{\delta}{\alpha} \hat{g}_t - (1-\sigma)\hat{\psi}_t - (1+\eta)\hat{z}_t + \hat{\tau}_t \right] \\
\Rightarrow \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \frac{(1-\omega\beta)(1-\omega)(\eta\alpha + \sigma)}{\omega\alpha} \hat{y}_t - \frac{\sigma\delta(1-\omega\beta)(1-\omega)}{\omega\alpha} \hat{g}_t \\
&\quad - \frac{(1-\sigma)(1-\omega\beta)(1-\omega)}{\omega} \hat{\psi}_t - \frac{(1+\eta)(1-\omega\beta)(1-\omega)}{\omega} \hat{z}_t + \frac{(1-\omega\beta)(1-\omega)}{\omega} \hat{\tau}_t \\
\Rightarrow \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa_1 \hat{y}_t - \kappa_2 \hat{g}_t - \kappa_3 \hat{\psi}_t - \kappa_4 \hat{z}_t + \kappa_5 \hat{\tau}_t
\end{aligned}$$

³² Remember that ψ has an average of 1 and, hence, it's steady state value is 1, i.e. $\psi^{SS} = 1$. Also, from equation 1.6, $W^{SS}(1-\tau^{SS}) = \chi(N^{SS})^\eta / (C^{SS})^{-\sigma}$.

³³ See appendix A.6.

A.11. The Model Solved

Plugging (2.8) into (2.1) and remembering that the expected values of the exogenous stochastic processes are 0 gives the following results for equation (2.1):

$$\begin{aligned}\hat{y}_t &= E_t \hat{y}_{t+1} - \gamma_1 (\lambda_y \hat{y}_t + \lambda_\pi^* \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + \gamma_2 \hat{g}_t + \gamma_3 \hat{\psi}_t \\ \Rightarrow (1 + \gamma_1 \lambda_y) \hat{y}_t &= E_t \hat{y}_{t+1} - \gamma_1 \lambda_\pi^* \hat{\pi}_t + \gamma_1 E_t \hat{\pi}_{t+1} + \gamma_2 \hat{g}_t + \gamma_3 \hat{\psi}_t\end{aligned}\quad (\text{A.25})$$

Plugging (2.11) and (2.12) into (A.25) and using the information in (2.9) and (2.10) gives:

$$\begin{aligned}(1 + \gamma_1 \lambda_y) (b_1 \hat{g}_t + b_2 \hat{c}_t + b_3 \hat{\psi}_t + b_4 \hat{z}_t) &= -\gamma_1 \lambda_\pi^* (c_1 \hat{g}_t + c_2 \hat{c}_t + c_3 \hat{\psi}_t + c_4 \hat{z}_t) \\ &+ \gamma_2 \hat{g}_t + \gamma_3 \hat{\psi}_t \\ \Rightarrow [(1 + \gamma_1 \lambda_y) b_1 + \gamma_1 \lambda_\pi^* c_1 - \gamma_2] \hat{g}_t & \\ + [(1 + \gamma_1 \lambda_y) b_2 + \gamma_1 \lambda_\pi^* c_2] \hat{c}_t & \\ + [(1 + \gamma_1 \lambda_y) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3] \hat{\psi}_t & \\ + [(1 + \gamma_1 \lambda_y) b_4 + \gamma_1 \lambda_\pi^* c_4] \hat{z}_t &= 0\end{aligned}\quad (\text{A.26})$$

A sufficient condition for (A.26) to hold is the following:

$$\begin{aligned}(1 + \gamma_1 \lambda_y) b_1 + \gamma_1 \lambda_\pi^* c_1 - \gamma_2 &= 0 \\ (1 + \gamma_1 \lambda_y) b_2 + \gamma_1 \lambda_\pi^* c_2 &= 0 \\ (1 + \gamma_1 \lambda_y) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3 &= 0 \\ (1 + \gamma_1 \lambda_y) b_4 + \gamma_1 \lambda_\pi^* c_4 &= 0\end{aligned}\quad (\text{A.27})$$

Using the information in (2.11) and (2.12) and remembering that the expected values of the exogenous stochastic processes are 0, (2.3) becomes:

$$\begin{aligned}c_1 \hat{g}_t + c_2 \hat{c}_t + c_3 \hat{\psi}_t + c_4 \hat{z}_t & \\ = \kappa_1 (b_1 \hat{g}_t + b_2 \hat{c}_t + b_3 \hat{\psi}_t + b_4 \hat{z}_t) - \kappa_2 \hat{g}_t - \kappa_3 \hat{\psi}_t - \kappa_4 \hat{z}_t + \kappa_5 \hat{c}_t & \\ \Rightarrow (c_1 - \kappa_1 b_1 + \kappa_2) \hat{g}_t + (c_2 - \kappa_1 b_2 - \kappa_5) \hat{c}_t & \\ + (c_3 - \kappa_1 b_3 + \kappa_3) \hat{\psi}_t + (c_4 - \kappa_1 b_4 + \kappa_4) \hat{z}_t &= 0\end{aligned}\quad (\text{A.28})$$

A sufficient condition for (A.28) to hold is the following:

$$\begin{aligned}c_1 - \kappa_1 b_1 + \kappa_2 &= 0 \\ c_2 - \kappa_1 b_2 - \kappa_5 &= 0 \\ c_3 - \kappa_1 b_3 + \kappa_3 &= 0 \\ c_4 - \kappa_1 b_4 + \kappa_4 &= 0\end{aligned}\quad (\text{A.29})$$

Using the first equations in (A.27) and (A.29) gives the following equation system and results for b_I and c_I :

$$\begin{aligned} (1 + \gamma_1 \lambda_y) b_1 + \gamma_1 \lambda_\pi^* c_1 - \gamma_2 &= 0 \\ c_1 - \kappa_1 b_1 + \kappa_2 &= 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow b_1 &= \frac{\gamma_1 \lambda_\pi^* \kappa_2 + \gamma_2}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \\ \Rightarrow c_1 &= \frac{\kappa_1 \gamma_2 - \kappa_2 (1 + \gamma_1 \lambda_y)}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \end{aligned}$$

Using the second equations in (A.27) and (A.29) gives the following equation system and results for b_2 and c_2 :

$$\begin{aligned} (1 + \gamma_1 \lambda_y) b_2 + \gamma_1 \lambda_\pi^* c_2 &= 0 \\ c_2 - \kappa_1 b_2 - \kappa_5 &= 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow b_2 &= -\frac{\kappa_5 \gamma_1 \lambda_\pi^*}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \\ \Rightarrow c_2 &= \frac{\kappa_5 (1 + \gamma_1 \lambda_y)}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \end{aligned}$$

Using the third equations in (A.27) and (A.29) gives the following equation system and results for b_3 and c_3 :

$$\begin{aligned} (1 + \gamma_1 \lambda_y) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3 &= 0 \\ c_3 - \kappa_1 b_3 + \kappa_3 &= 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow b_3 &= \frac{\gamma_3 + \kappa_3 \gamma_1 \lambda_\pi^*}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \\ \Rightarrow c_3 &= \frac{\kappa_1 \gamma_3 - \kappa_3 (1 + \gamma_1 \lambda_y)}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \end{aligned}$$

Using the fourth equations in (A.27) and (A.29) gives the following equation system and results for b_4 and c_4 :

$$\begin{aligned} (1 + \gamma_1 \lambda_y) b_4 + \gamma_1 \lambda_\pi^* c_4 &= 0 \\ c_4 - \kappa_1 b_4 + \kappa_4 &= 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow b_4 &= \frac{\kappa_4 \gamma_1 \lambda_\pi^*}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \\ \Rightarrow c_4 &= -\frac{\kappa_4 (1 + \gamma_1 \lambda_y)}{1 + \gamma_1 (\lambda_y + \lambda_\pi^* \kappa_1)} \end{aligned}$$

A.12. The Model Solved for Setting 3

Plugging (2.8) into (2.1), using the conditions in (2.6) and the information in (2.9) and (2.10) gives the following results for equation (2.1):

$$\begin{aligned}\hat{y}_t &= E_t \hat{y}_{t+1} - \gamma_1 (\lambda_y \hat{y}_t + \lambda_\pi^* \hat{\pi}_t - E_t \hat{\pi}_{t+1}) - \gamma_2 \phi (E_t \hat{y}_{t+1} - \hat{y}_t) + \gamma_3 \hat{\psi}_t \\ \Rightarrow (1 + \gamma_1 \lambda_y - \gamma_2 \phi) \hat{y}_t &= E_t \hat{y}_{t+1} - \gamma_1 \lambda_\pi^* \hat{\pi}_t + \gamma_1 E_t \hat{\pi}_{t+1} - \gamma_2 \phi E_t \hat{y}_{t+1} + \gamma_3 \hat{\psi}_t\end{aligned}\quad (\text{A.30})$$

Plugging (2.22) and (2.23) into (A.30) and using the information in (2.9) and (2.10) gives:

$$\begin{aligned}(1 + \gamma_1 \lambda_y - \gamma_2 \phi) (b_3 \hat{\psi}_t + b_4 \hat{z}_t) &= -\gamma_1 \lambda_\pi^* (c_3 \hat{\psi}_t + c_4 \hat{z}_t) + \gamma_3 \hat{\psi}_t \\ \Rightarrow [(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3] \hat{\psi}_t & \\ + [(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_4 + \gamma_1 \lambda_\pi^* c_4] \hat{z}_t &= 0\end{aligned}\quad (\text{A.31})$$

A sufficient condition for (A.31) to hold is the following:

$$\begin{aligned}(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3 &= 0 \\ (1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_4 + \gamma_1 \lambda_\pi^* c_4 &= 0\end{aligned}\quad (\text{A.32})$$

Using the information in (2.22) and (2.23), the conditions in (2.6) and the information in (2.9) and (2.10) gives the following results for (2.3):

$$\begin{aligned}c_3 \hat{\psi}_t + c_4 \hat{z}_t &= (\kappa_1 - \kappa_2 \phi) (b_3 \hat{\psi}_t + b_4 \hat{z}_t) - \kappa_3 \hat{\psi}_t - \kappa_4 \hat{z}_t \\ \Rightarrow (c_3 - (\kappa_1 - \kappa_2 \phi) b_3 + \kappa_3) \hat{\psi}_t + (c_4 - (\kappa_1 - \kappa_2 \phi) b_4 + \kappa_4) \hat{z}_t &= 0\end{aligned}\quad (\text{A.33})$$

A sufficient condition for (A.33) to hold is the following:

$$\begin{aligned}c_3 - (\kappa_1 - \kappa_2 \phi) b_3 + \kappa_3 &= 0 \\ c_4 - (\kappa_1 - \kappa_2 \phi) b_4 + \kappa_4 &= 0\end{aligned}\quad (\text{A.34})$$

Using the first equations in (A.32) and (A.34) gives the following equation system and results for b_3 and c_3 :

$$\begin{aligned}(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3 &= 0 \\ c_3 - (\kappa_1 - \kappa_2 \phi) b_3 + \kappa_3 &= 0 \\ \Rightarrow b_3 &= \frac{\gamma_1 \lambda_\pi^* \kappa_3 + \gamma_3}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi) \lambda_\pi^*) - \gamma_2 \phi} \\ \Rightarrow c_3 &= \frac{(\kappa_1 - \kappa_2 \phi) \gamma_3 - \kappa_3 (1 + \gamma_1 \lambda_y - \gamma_2 \phi)}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi) \lambda_\pi^*) - \gamma_2 \phi}\end{aligned}$$

Using the second equations in (A.32) and (A.34) gives the following equation system and results for b_4 and c_4 :

$$\begin{aligned}
(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_4 + \gamma_1 \lambda_\pi^* c_4 &= 0 \\
c_4 - (\kappa_1 - \kappa_2 \phi) b_4 + \kappa_4 &= 0 \\
\Rightarrow b_4 &= \frac{\gamma_1 \lambda_\pi^* \kappa_4}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi) \lambda_\pi^*) - \gamma_2 \phi} \\
\Rightarrow c_4 &= -\frac{\kappa_4 (1 + \gamma_1 \lambda_y - \gamma_2 \phi)}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi) \lambda_\pi^*) - \gamma_2 \phi}
\end{aligned}$$

A.13. The Model Solved for Setting 4

Plugging (2.8) into (2.1), using the conditions in (2.7) and the information in (2.9) and (2.10) gives the following results for equation (2.1):

$$\begin{aligned}
\hat{y}_t &= E_t \hat{y}_{t+1} - \gamma_1 (\lambda_y \hat{y}_t + \lambda_\pi^* \hat{\pi}_t - E_t \hat{\pi}_{t+1}) - \gamma_2 \phi (E_t \hat{y}_{t+1} - \hat{y}_t) + \gamma_3 \hat{\psi}_t \\
\Rightarrow (1 + \gamma_1 \lambda_y - \gamma_2 \phi) \hat{y}_t &= E_t \hat{y}_{t+1} - \gamma_1 \lambda_\pi^* \hat{\pi}_t + \gamma_1 E_t \hat{\pi}_{t+1} - \gamma_2 \phi E_t \hat{y}_{t+1} + \gamma_3 \hat{\psi}_t \quad (\text{A.35})
\end{aligned}$$

Plugging (2.22) and (2.23) into (A.35) and using the information in (2.9) and (2.10) gives:

$$\begin{aligned}
(1 + \gamma_1 \lambda_y - \gamma_2 \phi) (b_3 \hat{\psi}_t + b_4 \hat{z}_t) &= -\gamma_1 \lambda_\pi^* (c_3 \hat{\psi}_t + c_4 \hat{z}_t) + \gamma_3 \hat{\psi}_t \\
\Rightarrow [(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3] \hat{\psi}_t & \\
+ [(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_4 + \gamma_1 \lambda_\pi^* c_4] \hat{z}_t &= 0 \quad (\text{A.36})
\end{aligned}$$

A sufficient condition for (A.36) to hold is the following:

$$\begin{aligned}
(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3 &= 0 \\
(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_4 + \gamma_1 \lambda_\pi^* c_4 &= 0 \quad (\text{A.37})
\end{aligned}$$

Using the information in (2.22) and (2.23), the conditions in (2.7) and the information in (2.9) and (2.10) gives the following results for (2.3):

$$\begin{aligned}
c_3 \hat{\psi}_t + c_4 \hat{z}_t &= (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) (b_3 \hat{\psi}_t + b_4 \hat{z}_t) - \kappa_3 \hat{\psi}_t - \kappa_4 \hat{z}_t \\
\Rightarrow (c_3 - (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) b_3 + \kappa_3) \hat{\psi}_t &+ (c_4 - (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) b_4 + \kappa_4) \hat{z}_t = 0 \quad (\text{A.38})
\end{aligned}$$

A sufficient condition for (A.38) to hold is the following:

$$\begin{aligned}
c_3 - (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) b_3 + \kappa_3 &= 0 \\
c_4 - (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) b_4 + \kappa_4 &= 0 \quad (\text{A.39})
\end{aligned}$$

Using the first equations in (A.37) and (A.39) gives the following equation system and results for b_3 and c_3 :

$$\begin{aligned}
(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_3 + \gamma_1 \lambda_\pi^* c_3 - \gamma_3 &= 0 \\
c_3 - (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) b_3 + \kappa_3 &= 0
\end{aligned}$$

$$\begin{aligned}\Rightarrow b_3 &= \frac{\gamma_1 \lambda_\pi^* \kappa_3 + \gamma_3}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \lambda_\pi^*) - \gamma_2 \phi} \\ \Rightarrow c_3 &= \frac{(\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \gamma_3 - \kappa_3 (1 + \gamma_1 \lambda_y - \gamma_2 \phi)}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \lambda_\pi^*) - \gamma_2 \phi}\end{aligned}$$

Using the second equations in (A.37) and (A.39) gives the following equation system and results for b_4 and c_4 :

$$\begin{aligned}(1 + \gamma_1 \lambda_y - \gamma_2 \phi) b_4 + \gamma_1 \lambda_\pi^* c_4 &= 0 \\ c_4 - (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) b_4 + \kappa_4 &= 0 \\ \Rightarrow b_4 &= \frac{\gamma_1 \lambda_\pi^* \kappa_4}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \lambda_\pi^*) - \gamma_2 \phi} \\ \Rightarrow c_4 &= -\frac{\kappa_4 (1 + \gamma_1 \lambda_y - \gamma_2 \phi)}{1 + \gamma_1 (\lambda_y + (\kappa_1 - \kappa_2 \phi + \kappa_5 \theta) \lambda_\pi^*) - \gamma_2 \phi}\end{aligned}$$

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