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The Economics of Sleeping

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Abstract

Sleep is a source of energy. This energy is available in limited quantity and individuals must decide when it should be renewed and when it should be consumed. The economics of sleeping and the economics of resource extraction are one and the same. More specifically, utility maximization with respect to sleep satisfies Hotelling's rule on the optimal utilization of natural resources. Several applications emerge from the analysis. These include the effects of labor-market opportunities on sleep patterns; the effect of having children; the consequences of the decreased division of labor within the household; and the relationship between sleep deprivation and obesity.

JEL Classification: D11, D13, I12, J12, J13, J22, J24, Q20
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Sleep has vast implications for multiple aspects of our lives. It consumes a larger amount of our scarce time than any other single activity. Despite this, economists' extensive efforts at describing the optimal usage of time have largely ignored decisions about sleep, as if the supply of waking time is of fixed quality and quantity. This is surprising, as the willing control of sleep seems to be quite extensive. Furthermore, scientists in multiple fields have described many ways in which sleep is related to cognitive performance, alertness, memory, decision making, reasoning, problem solving and accidents (Van Dongen et al. 2003; Turner et al. 2007). Those aspects are surely important for work behavior and productivity.

The first paper to address this topic was by Biddle and Hamermesh (1990) who found that the length of sleep varies between individuals so that higher wage rates reduce sleep time among men. In their formulation, sleep is assumed to generate utility in addition to having a positive effect on wages. In this paper we extend their contribution by modeling the decision to sleep as an investment decision in the level of alertness that we enjoy during the day. In so doing we introduce the quality of sleep and the rate at which one tires during the day as two factors that can be influenced by individuals in multiple ways. Our formulation shows that the economics of sleeping resembles the economics of resource extraction so that the utility-maximizing conditions for sleeping are equivalent to Hotelling's rule for the optimal utilization of resources, which is the maximization of the scarcity rent (see Hotelling, 1931).

Sleeping and resting make us alert and can enhance the experiences of both work and leisure. The alertness we get from resting can in this way be viewed as a resource. There is a clear inter-temporal tradeoff: longer sleep makes us feel better while awake, at least up to a point, but it also makes the time spent awake shorter. Clearly, some people will choose to sleep more than others and these people will have shorter days. Time management can in this way be studied as the optimal utilization of a resource, which is the alertness that we get from resting. Just as the optimal harvesting of a forest or a fish stock can be studied with the tools of economics, so can the optimal sleep pattern. In this way the allocation of time to sleep can be modeled in a broad sense that can encompass other applications of specific sleep related situations.[1] With regard to the human-capital literature, it is also possible to view sleeping as human-capital production, either directly or as an input into health production, following Grossman (1972).

[1] Our general model can encompass the more specific models of sleep that are to be found in the literature. Those include sleep quality (Yaniv, 2004) and the analysis of sleep as an input in health production (Contoyannis and Jones, 2004).

Individuals resort to different methods to time-compress the benefits of sleep or slow the onset of fatigue during waking time. These include the use of sleep medication that make sleeping more productive, pillows, mattresses, time-saving gadgets, outsourcing of what used to be home production and so on. Expenditures in markets for specific sleep aids in the United States have been estimated in billions of dollars (Walsh and Engelhardt, 1999). Alcoholic beverages aid relaxation and are commonly used as a sleep aid, although they can disrupt sleep throughout the night and prevent the deeper stages of sleep from occurring (Wyatt et al. 1999). The use of different drugs that provide stimulus during the day such as caffeine, nicotine, and in extreme cases amphetamines and cocaine all provide alertness and delay the onset of fatigue. This may also be true of food in a more general sense. The extensive implications of sleep for individuals and society as a whole are thus apparent. This paper adds to the literature by conceptualizing sleep decisions in a general way, while still allowing for sufficient detail for more specific applications.

1. Background

Sleep has a restorative function and regular sleep is essential for the human survival. Although sleep has seldom been the focus of economic analysis, sleep has been the subject of intense research in other fields. Sleep timing depends on several factors, of which willed behavior, the focus of the current analysis, is one. However, this is not to suggest that physiological aspects are not important. Sleep propensity is a function of messages from the circadian clock, an inner time-keeping device that tells the body it needs to sleep. The circadian clock affects many of the bodily processes that are associated with wakefulness. In humans sleepiness occurs as the circadian element causes the release of the hormone melatonin (Wyatt et al. 1999).

The optimal amount of sleep depends on several factors and is partly biologically determined. For example, the timing of sleep in relation to an individual's circadian rhythms affects the quality of sleep, where a sleep episode is relatively inefficient if it occurs during the "wrong" time of day (Wyatt et al. 1999; Aeschbach et al. 2003; Van Dongen Vitellaro and Dinges 2005). However, the data also suggest that sleep patterns vary significantly across human cultures and demographic groups (Webb 1985; Biddle and Hamermesh 1990; Szalontai 2006). The reasons are not well understood and could very well be determined by willed behavior due to varying incentives.[2] In short, sleep and its effects vary a great deal with

[2] Surprisingly, the effects on health are not clear as several studies have shown sleep duration of more than seven to eight hours per day to be associated with increased mortality. Causal links are currently speculative: the

factors such as the presence of children, diet, appetite, income, and both the nature and amount of work. Each one will be described in more detail below.

2. The basic idea

We assume that utility depends on daytime alertness *A*, as described by the following equation

$$U = \log(A) \tag{1}$$

Our alertness is enjoyed during the time spent awake *t*, which is the difference between the total time available in one day t_1 (24 hours if the day is the relevant period) and the time we spend sleeping t_0.

$$t = t_1 - t_0 \tag{2}$$

We assume that the relationship between the time spent awake and the value of *A* in a given day is the following

$$A = A_0 e^{at} = A_0 e^{a(t_1 - t_0)} \tag{3}$$

where *a* is a positive function of t_0, $\partial a / \partial t_0 = a_{t_0} > 0$, and the second derivative is negative, $a_{t_0 t_0} < 0$. This creates a clear tradeoff as sleeping longer hours makes us more alert during the day while reducing waking time. The first-order condition for utility maximization with respect to t_0 is

$$-a + (t_1 - t_0) a_{t_0} = 0 \tag{4}$$

Note that the first term denotes the fall in utility from lost waking time and the second term has the gain from being more alert during the day. This can be written as

$$a = (t_1 - t_0) a_{t_0} \tag{4'}$$

where the left-hand side has the marginal cost of sleeping and the right-hand side has the marginal benefit of sleeping.[3]

The analysis so far gives the flavor of our reasoning. However, it is limited in that there are very few exogenous variables and we can generate only a limited range of comparative

available data may reflect such things as depression, socioeconomic status, or even alcohol use, as these studies can only be used to determine correlation, but not causation (Patel et al. 2004 and 2006a; Ferrie et al. 2007; Born, Rasch, and Gais 2006).

[3] The second-order condition is $-2a_{t_0} + (t_1 - t_0) a_{t_0 t_0} < 0$.

statics predictions. There is really only one prediction, that having more time t_1 will make us sleep longer t_0, as seen from equation (4');

$$t_0 = t_1 - \frac{a}{a_{t_0}} \qquad (4'')$$

But note that this is not a reduced-form solution because both a and a_{t_0} depend on the value of t_0. In order to generate more testable predictions we now model the effects of different sleep-related parameters.

3. More on the optimal length of sleep

Our level of alertness is not constant during the day, we gradually tire and end the day less alert. Similarly, we gradually acquire alertness during the night. To model this we rewrite equations (1) and (3) so that they capture alertness at a given moment t where t_0 is the time spent sleeping and t_1 is the total time available in one day. The utility function then becomes

$$u_t = \log(A_t) = \log\left(A_0 e^{st_0} e^{-f(t-t_0)}\right) \qquad (5)$$

where A_0 denotes how alert we feel in the absence of sleep (or in some sense at the beginning of a sleep episode); s is the rate at which we acquire alertness while sleeping or the quality of sleep; and f is the rate at which we exhaust alertness or tire during the day. Utility depends on $A_{0,}$ but also on how well we rest while sleeping as measured by s, how long we sleep as measured by t_0, the rate at which we exhaust during the day as measured by f and how long we stay up as measured by t_1-t_0. Taking logs gives

$$u_t = a_0 + st_0 - f(t - t_0) \qquad (6)$$

The utility the individual gets in a day is given by the integral below, where we assume that the act of sleeping gives no contemporaneous utility.

$$U = \int_{t_0}^{t_1} \left(a_0 + st_0 - f(t - t_0)\right) dt \qquad (7)$$

Now taking the derivative with respect to t_0 and using Leibnitz's rule gives the first-order condition for utility maximization with respect to sleeping t_0;

$$a_0 + st_0 = \int_{t_0}^{t_1} (s + f)\, dt \qquad (8)$$

The left-hand side has the marginal cost in terms of utility lost during the time of additional sleep, a_0+st_0, and the right-hand side is the marginal benefit in terms of feeling more alert while awake. The marginal benefit of sleeping longer hours consists of two parts: First, someone sleeping longer hours will feel more rested during the day and, second, he will not feel as tired at the end of the day because he has not stayed up as long.

Equation (8) is a version of Hotelling's rule (1931) for the maximization of the scarcity rent from a resource – that the percentage increase in the value of the resource per unit of time should equal the rate of interest. Evaluating the integral gives,

$$a_0 + st_0 = (s+f)(t_1 - t_0) \tag{9}$$

Appealing to Hotelling, we may characterize the act of getting out of bed as initiating "resource extraction," which gives us benefits in terms of utility, while remaining in bed and resting is tantamount to not extracting but letting the resource become more valuable instead. Interpret the left-hand side of the equation not as the marginal cost of sleeping but as the marginal benefit of waking up, that is sleeping less, and the right-hand side is the marginal cost (in alertness) of waking up in the morning. The marginal benefit consists of the utility we can enjoy during the extra time that we now spend awake while the marginal cost consists of the sacrifice in terms of alertness – lower utility – during the waking time. The benefit is what we gain from "extraction," which is utility during the time we do not sleep, while the cost consists of the resource not becoming more valuable – not resting any further – which is manifested in being less alert during the day.

If we divide through equation (9) by U we get the following:

$$\frac{a_0 + st_0}{U} = \frac{(s+f)(t_1 - t_0)}{U} \tag{10}$$

The left-hand side is an "interest rate," that is the gain from extracting the resource – getting out of bed! – divided by the value of the resource U, while the right-hand side is the rate of increase of the value of the resource if not extracted – which is feeling even more rested and anticipating a shorter day, which brings less fatigue – again divided by the value of the resource U. We could rewrite the equation in the familiar Hotelling formulation

$$r = \frac{\dot{U}}{U} \tag{11}$$

where the rise in the value of the resource is denoted by $\dot{U}$ and r is the rate of return to extraction.

From the equality of the marginal cost and benefit of sleeping in equation (9) we retrieve the optimal length of sleep t_0*,

$$t_0^* = \frac{(s+f)t_1 - a_0}{2s+f} \tag{12}$$

which then gives the optimal level of alertness during the day.

$$A_0 \exp\left(s\frac{(s+f)t_1 - a_0}{2s+f} - f\left(t - \frac{(s+f)t_1 - a_0}{2s+f} \right) \right) \tag{13}$$

People's productivity or mental alertness at any time t is thus made endogenous and depends on the choices people make when maximizing their utility. In this way we find that our alertness during the day depends on various parameters, such as s, f, t_1 and a_0. We explore this dependence further below, but first we introduce the distinction between work and leisure.

4. Work-leisure choice added

In this chapter we add to the model by introducing labor-market choices and showing how our model can help explain different phenomena observed in labor markets. We introduce time spent at work and possible interactions between labor-market behavior, sleep and wages.

4.1 The basic idea

Define X as a good purchased in the market at a price of \$1. Let w be the wage rate, h be hours of work and assume no property income so that $X = wh$ and $t = t_1 - t_0 - h$. Assume the following utility function, which has two elements

$$U = U\left(a\left(t_1 - t_0 - h\right), wh\right) \tag{14}$$

The first-order condition for t_0 is

$$a = \left(t_1 - t_0 - h\right)a_{t_0} \tag{15}$$

which is similar to equation (4) above. There is a new necessary condition for h, which is

$$-aU_1 + wU_2 = 0 \tag{16}$$

or

$$\frac{U_1}{U_2} = \frac{w}{a} \tag{17}$$

The last equation equates the marginal rate of substitution between A and X (U_1/U_2) to the shadow price of A (w/a). Note that a is the marginal product of t in the production of A because a now depends on t_0, but t_0 is held constant when optimal h is selected.

The model with work differs from the one developed by Biddle and Hamermesh (1990) because they allow sleep to affect the wage rate and to have a direct positive impact on utility. They do not allow sleep to affect the productivity of nonmarket time. Now assume that sleep has a positive impact on w. The first-order condition for h is not changed since t_0 is held constant when optimal h is selected. The first-order condition for t_0 becomes

$$aU_1 = a_{t_0}U_1\left(t_1 - t_0 - h\right) + U_2 w_{t_0} h \tag{18}$$

where the right-hand side has the marginal benefit of sleeping and the left-hand side the marginal cost. Rearranging gives

$$\frac{U_1}{U_2} = \frac{w}{a} = \frac{w_{t_0} h}{a - a_{t_0}\left(t_1 - t_0 - h\right)}. \tag{19}$$

It follows that the individual will decide to sleep longer when sleep raises his wages in the labor market.

4.2 Digging deeper

In the basic idea expressed in the previous section, higher wages w will never have the effect of making individuals reduce their sleep. Higher wages will only have the effect of making them cut into their leisure hours. However, we can augment the model of Section 3 by introducing the work-leisure choice. Assume that every moment spent awake is split between leisure and work. We thus sometimes focus exclusively on our work, sometimes only on leisure but also sometimes on both. We augment equation (5) by distinguishing between the shares of each moment spent awake devoted to leisure l and to labor 1-l. The utility function now becomes,

$$u_t = \log\left(A_0 e^{st_0 - f(t - t_0)}\left(w(1-l)\right)^{\alpha} l^{1-\alpha}\right) \tag{20}$$

where w denotes the real wage. Note that alertness affects one's ability to both work and enjoy leisure. Our measure of alertness is hence a measure of workers' productivity, which we endogenise by deriving an optimality condition for the proportion of the day spent sleeping. The more rested one feels, the greater is productivity at work and the greater one enjoys leisure. Taking logs gives,

$$u_t = a_0 + st_0 - f\,(t - t_0) + \alpha \log\big(w(1-l)\big) + (1-\alpha)\log(l) \tag{21}$$

Repeating the derivation from Section 3 we now take the integral of the utility function over the day;

$$U = \int_{t_0}^{t_1} \Big[a_0 + st_0 - f\,(t - t_0) + \alpha \log\big(w(1-l)\big) + (1-\alpha)\log(l) \Big] dt \tag{22}$$

The first-order conditions with respect to t_0 and l follow. First set the derivative with respect to t_0 equal to zero;

$$a_0 + st_0 + \alpha \log\big(w(1-l)\big) + (1-\alpha)\log(l) = (s + f)(t_1 - t_0) \tag{23}$$

The left-hand side again has the marginal costs of sleeping, which is comprised of the lost utility of income and leisure during additional hours of sleep. The right-hand side has the marginal utility of sleeping – that is the greater alertness one has during the day. Note that a higher wage w adds to the marginal cost of sleeping. The first-order condition with respect to l follows;

$$l = 1 - \alpha \tag{24}$$

where the share of waking time devoted to leisure is equal to 1-α. From (23) we get an equation for the optimal level of t_0, analogous to equation (12) above;

$$t_0^* = \frac{(s + f)t_1 - a_0 - \alpha \log\big(w(1-l)\big) - (1-\alpha)\log(l)}{2s + f} \tag{25}$$

Note that the optimal time spent sleeping is decreasing in the wage w.

4.3 Efficiency wages

We have not modeled wage setting so far. However, firms may take the effect of wages on sleep and alertness into account when setting wages. We have found that the higher the wage, the earlier it becomes optimal to wake up in the morning because the opportunity cost of sleeping is greater.

Assume a log production function where output depends on the number of workers L, the alertness of each worker A and technology Λ. The instantaneous profits π are then given by the following equation.

$$\pi = \Lambda \log(AL) - wL \tag{26}$$

The representative firm maximizes daily profits with respect to wages w and employment L. While the first-order condition with respect to L takes the trivial form $L=\Lambda/w$, maximizing with respect to wages turns out to be more interesting because wages affect the time spent awake as well as the time spent working.

The alertness of a worker depends on the amount of sleep he gets t_0;

$$A = A_0 e^{st_0 - f(t-t_0)} \tag{27}$$

and the firm maximizes daily profits π, which is the difference between output and the wage bill during the time spent at work $\alpha(t_1-t_0)$:

$$\pi = \alpha \int_{t_0}^{t_1} \left(\Lambda \log\left(A_0 e^{st_0 - f(t-t_0)} L \right) - wL \right) dt \tag{28}$$

Taking logs gives

$$\pi = \alpha \int_{t_0}^{t_1} \left[\Lambda \left(a_0 + st_0^* - f\left(t - t_0^*\right) + \log(L) \right) - wL \right] dt \tag{29}$$

where the optimal length of sleep has been found in equation (25). From the equation above we find that the higher the wage, the shorter is the optimal sleep;

$$\frac{dt_0^*}{dw} = -\frac{\alpha}{(2s+f)w} < 0 \tag{30}$$

Maximizing profits with respect to w and taking into account the equation for the optimal length of sleep gives

$$\frac{\alpha\Lambda}{(2s+f)w} \left[\underbrace{a_0 + st_0^* + \log(L) - wL}_{I} - \underbrace{(s+f)\left(t_1 - t_0^*\right)}_{II} \right] - \underbrace{\alpha L \left(t_1 - t_0^*\right)}_{III} = 0 \tag{31}$$

The first terms in the square bracket denote the effects of a longer working day; by paying higher wages the firm induces workers to work more hours which contributes to profits. The second term has the loss due to workers being less alert during the day and this lowers profits. The final term shows the increase costs due to higher wage rates. The marginal benefit of raising the wage thus consists of the extra time that workers put in when they are being paid more while the marginal cost consists of lost output due to greater fatigue during the day, as well as higher wage costs.[4] Note that the optimal wage is increasing in the level of

[4] The second-order condition is satisfied;

$$\frac{d^2\pi}{dw^2} = -\frac{\alpha}{(2s+f)w^2} t_0^* - \frac{\alpha^2}{(2s+f)w} = -\frac{\alpha}{(2s+f)w}\left(\frac{t_0^*}{w} + \alpha \right) < 0 .$$

technology A; it is increasing in the share of waking time spent working α; it is increasing in the ability of workers to go without sleep a_0; and it is decreasing in the rate at which workers tire during the day f; as well as in the quality of sleep s.[5]

5. Further applications

In this chapter we will give a few examples of how the insights of our model help understand different labor market phenomena, such as the wage distribution and self-assessed happiness.

5.1 Human capital

It is a well-established fact that individuals differ in terms of the sleep that they need (Aeschbach 2003; Van Dongen Vitellaro and Dinges 2005). Our model implies that not needing much sleep is a form of human capital. The less sleep we need, the less we choose to sleep and the more time we have for work and leisure. Taking the derivative of equations (12) and (25) gives

$$\frac{dt_0}{da_0} = -\frac{1}{2s+f} < 0 \tag{32}$$

The intuition is simply that if we can do without much sleep, that is be productive at work and enjoy our leisure, then sleeping longer hours entails a greater sacrifice in terms of the time lost while sleeping (again, imputing no utility to sleeping time other than accretions to alertness).

5.2 Burn out

Our model implies that high wages can be a double-edged sword. High wages bring more consumption, which gives utility. But high wages also raise the opportunity cost of sleeping as shown by the derivative of (25)

$$\frac{dt_0}{dw} = -\frac{\alpha / w}{2s+f} < 0 \tag{33}$$

With a higher wage w the opportunity cost of sleeping longer hours is raised, which incentives us to wake up earlier in the morning, the more so the more we like income as expressed by α. Moreover, the impact decreases in w due to the diminishing marginal utility of money income. It follows that high wages have the effect of reducing people's alertness, which has the effect of lowering their average instantaneous productivity while presumably raising their production per day.

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[5] We can ignore the indirect effect of these changes going through t_0 by appealing to the envelope theorem.

5.3 Children

One effect of having children is to shorten the total time available for work, leisure and sleep, which is represented by t_1 in the model.[6] This can come about when significant time is spent on child rearing and other house chores that are essential for the upbringing of a child. The lower value of t_1 will reduce the optimal sleeping time. This is obvious in terms of equations (15) and (18) and we can also show this by using equations (12) and (25) by taking the derivative shown below:

$$\frac{dt_0}{dt_1} = \frac{s+f}{2s+f} > 0 \tag{34}$$

Note that the derivative is less than one so that people optimally respond to having less time by both sleeping shorter hours as well as by devoting less time to work and leisure. The marginal benefit of sleeping is smaller than before when t_1 falls because we have less time to "enjoy" being alert and this makes us sleep less. As a result a worker's productivity (see equation (13)) is reduced. The effect on productivity is given by the following equation, which is derived from equations (13) and (34) above.

$$\frac{dA_t}{dt_1} = A_0 \frac{(s+f)^2}{2s+f} \exp\left(s\frac{(s+f)t_1 - a_0}{2s+f} - f\left(t - \frac{(s+f)t_1 - a_0}{2s+f} \right) \right) > 0 \tag{35}$$

Clearly, having less time during the day will affect our productivity adversely by making us sleep less at night.

These results are in accordance with the observation that parenthood may not be conducive to an increase in the general level of happiness (Layard, 2005) and the observation that parenthood may also have a negative effect on performance at work.[7] They are also in accordance with Becker (1965), who argued that one reason why women may have lower wages than men, other things equal, is that they carry a disproportionate responsibility for children. Becker claims that even if hours at work are held constant, the diversion of attention might be a reason to consider. This is along the same lines as the model presented here, in which energy consuming activities at home may induce us to sleep less.

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[6] In that raising one's own children is usually not remunerated, having children may be viewed as a form of leisure. In that children may support their parents at later stages of the life cycle, having children may be viewed as work (investment). For analytical purposes, time spent on children is however separated from time spent working and enjoying leisure in this analysis.

[7] The reader is directed to Waldfogel (1998) for a review of studies on how changes in family structure affect the labor market.

Another effect of having children is that young children may affect the quality of their parents' sleep. Using equation (12) we find that when the quality of sleep deteriorates we may respond by either sleeping more or less. Webb (1985) finds that children reduce sleep duration. However, Biddle and Hamermesh (1990) find that this effect is confined to women. A fall of *s* in our model will both decrease the marginal benefit from sleeping – because we enjoy our day less – as well as reducing the marginal cost – the last hour lost in sleep is worth less because we would have woken up less rested:

$$\frac{dt_0}{ds} = \frac{2a_0 - ft_1}{(2s+f)^2} < ? > 0 \tag{36}$$

The effect on productivity is however unambiguous. From equations (13) and (36) we get

$$\frac{dA_t}{ds} = A_0 \frac{2s(s+f)t_1 + fa_0}{(2s+f)^2} \exp\left(s\frac{(s+f)t_1 - a_0}{2s+f} - f\left(t - \frac{(s+f)t_1 - a_0}{2s+f} \right) \right) > 0 \tag{37}$$

When *s* falls our productivity is adversely affected. The inclusion of *s* in the model provides an opportunity to study other elements that affect *s* and the individuals' choices in that regard. Sleeping aids are but one example and this would allow a pathway for formulations along the lines of Yaniv (2004), who considered sleep decisions with a special focus on insomnia.

Yet another element of child rearing is that their presence may affect the rate at which one tires during waking hours, *f*. The following section discusses the effect of contextual influences on the rapidity of fatiguing and the individuals' options in that regard. The effect of children can thus be thought of in the context of the next section as well.

5.4 Gadgets

A recent article in *The Times* of London reports that "researchers have found that typical middle-class city dwellers now have so many timesaving gadgets that they can cram into 24 hours the same quantity of tasks that a decade ago would have taken 31 hours to complete."[8] One way to view this development is to say that people are saving time. The alternative way proposed in this paper is to say that people are trying to conserve their energy by having devices that help perform different tasks.

One can furthermore employ servants and devices that help with domestic chores so as to make life less draining. Examples include baby sitters who perform perhaps one of the most

[8] John Harlow, "Study finds we get 31 hours into a day: Multitasking make the modern man," *The Sunday Times*, April 13, 2008.

energy-consuming task; fast-food restaurants that help us avoid shopping for food and cooking; ready-made meals; cleaners who make our surroundings tidy; vacuum cleaners that make it easier to clean carpets; television that gives us insights into the life of others, news and information without exertion on our behalf; and computers that help us communicate with other people at close to zero costs in terms of effort and also to perform miscellaneous tasks such as shopping for everything from groceries to books to holidays; and the automobile which reduces the effort of going from one place to another.

We can use the equations above to show that the more rapidly we tire during the day, that is the higher the value of *f*, the more we decide to sleep:

$$\frac{dt_0}{df} = \frac{st_1 + a_0}{2s + f} > 0 \tag{38}$$

It follows that devices that help us conserve our energy during the day, the effects of which we can capture by a lower level of *f*, will have the effect of reducing the time spent sleeping. The effect on productivity can be derived from equation (13);

$$\frac{dA_t}{df} = A_0 \frac{\left(3s^2 + 4sf + f^2\right)t_1 - sa_0 - t\left(2s + f\right)}{\left(2s + f\right)^2} \exp\left(s\frac{\left(s + f\right)t_1 - a_0}{2s + f} - f\left(t - \frac{\left(s + f\right)t_1 - a_0}{2s + f} \right)\right) < ? > 0 \tag{39}$$

A lower value of *f* will reduce alertness and productivity in the morning – because we sleep less as shown in the equation above – while productivity may be higher in the evening.

However, gadgets are not the only way with which one can affect *f*. As the main focus of this paper is the deliberate control of sleep time, it is important to keep in mind other consumption goods with which one can slow the onset of fatigue. There are clearly stimulants, such as caffeine that affects the sleepiness-causing hormones (Wyatt et al. 1999). Maybe less obvious, but no less important, is the fact that increased food consumption can boost alertness. Sleeping and eating are thus related. In fact, the two acts can be regarded as compliments in the production of alertness, although sleep loss is associated with an increase in appetite that is excessive in relation to the caloric demands of extended wakefulness. Animals subjected to sleep deprivation increase their food intake considerably and studies in humans have shown that the levels of hormones that regulate appetite are profoundly influenced by sleep duration (Van Cauter et al. 2005). Thus the relationship between body weight and sleep can be considered in the context of a choice model with sleeping and consuming food being two alternative ways for energy replenishment.

Young children clearly have the effect of raising *f*. We have shown that the optimal response to this effect is to sleep longer hours – while the optimal response to having less time for work and leisure due to children in Section 5.3 was to sleep less – and sleeping longer has the effect of starting the day being more alert while the higher value of *f* makes us end the day feeling more tired.

5.5 Unproductive Time

A significant part of the day is often taken up by activities that are not conducive to either productivity or the enjoyment of leisure. Take commuting as an example. Clearly people who commute have made the choice of spending more time travelling to and from work in return for enjoying a bigger house or a cheaper one, hence having more to spend on consumption. These individuals can gain more consumption by spending time commuting – hence enjoying a lower rent or mortgage – than they can by instead spending this time working. But the time commuting depends on infrastructure, on traffic and other factors outside one's control. In this way, commuting cuts into the time they have available for work and play and we have shown that this would make them sleep less and so be less alert and productive during the day.

While commuting is to a certain extent a choice variable, a lot of time is also wasted during the day due to unintended developments. Queuing for service or to pay for purchases cuts into the effective time we have for work and leisure. Other rent-seeking activities have the same effect. Boring and useless conversations, difficult and rebellious colleagues, an inefficient public sector, meaningless political debate, television and vacuous news stories all have the effect of increasing unproductive time and hence cutting into our sleep and reducing our productivity and happiness.

5.6 Happiness

Our analysis supports the truism that people respond to enhanced opportunities for pleasure by sleeping less, as things that increase enjoyment during waking hours raise the opportunity cost of sleeping. A higher wage is always welcome, yet it raises the opportunity cost of sleeping and so makes us enjoy the day less on that account. This is consistent with cross-country results, which show sleep duration to be negatively related to per capita gross national income (Szalontai, 2006). We end up having more consumption but through the lack of sleep being in worse shape during the day to enjoy our income and our leisure.

Having children is perhaps life's most profound experience. However, the raising of children takes a lot of time and makes us adjust by sleeping less. This reduces our production, wages and the utility of leisure. If either the mother or the father takes on a disproportionate share of the responsibilities then she or he will be more affected.

Living in the modern world may be a blessing, but there is a secondary effect in that we may adjust by sleeping less and subsequently start the day feeling more lethargic. Having an ideal life by modern standards may make us sleep less and so feel less happy on account of not enjoying leisure or performing at work up to our previous standards. This is consistent with much of the happiness research, such as Layard (2005) who finds that increased income and the presence of children have limited, if any, effect on our self-reported level of happiness.

Solnick and Hemenway (1997) provide some empirical information about the relative and absolute value of various desiderata. They find that both absolute and relative position vis-à-vis one's perceived peers and near-peers matter to people. However, the importance of each one varies greatly across goods, with vacation time evoking the least concern for relative standing. Positional concerns were much stronger for income, but interestingly they were even sharper when it came to the attributes of one's children – attractiveness, intelligence and education. Although the authors did not include sleep duration in their examination, anecdotal evidence suggests concerns about relative standing to be even less important for sleep than it is for vacation time. It is thus possible that in order to improve relative standing with regard to income and child characteristics, one depletes a desideratum that is mainly valued in absolute terms by sleeping less. In doing so the individuals do not take into account the negative positional externality imposed on others. It is an empirical question whether this sheds some light on the Easterlin paradox of the limited relationship between GDP and self-assessed happiness and the perplexing results about parenthood's effect on happiness (Easterlin, 1973; Layard, 2005).

6. Conclusions

How much we sleep is to some extent an economic decision. We sleep longer hours if we have difficulty going without sleep; are less time constrained; our wages are low; and when we expect to have a tiring day. We may also respond to sleeping problems by sleeping fewer hours. This decision affects our alertness during the day, which affects our performance on the job as well as how much we enjoy leisure. However, sleeping less leaves us more time for work and play. There is a clear inter-temporal tradeoff between the length of waking time, on the one hand, and the quality of work and the enjoyment from leisure, on the other hand. Alertness is a resource that is gradually used up during the day and subsequently replenished at night. The question of how long we sleep is in essence a question on how we can optimally mine this resource. Our optimality condition is a form of Hotelling's rule, which says that the growth in alertness during sleeping hours should equal the "interest rate" which is the benefit we get from entering the world of waking activity - "extraction".

The economic intuition laid out in this paper has helped us cast a new light on various economic phenomena. People who by nature are able to remain alert without much sleep have a form of human capital that allows them to stay up longer each day. Raising children reduces the time we have left for work, leisure and sleep. The reduced sleep can subsequently decrease one's productivity in the workplace. When the mother (father) takes on a disproportionate share of the child-raising responsibilities, her (his) sleep is bound to be affected. Consequently her (his) productivity in the workplace, if she (he) chooses to have a job, is diminished. The male-female wage differential could in this way be partly explained by women taking on a larger share of the child-rearing responsibilities. The modern world offers many devices and gadgets that help us economize on our time and energy. By taking advantage of these gadgets we can better remain alert during the day. Our model implies that people will respond by sleeping less, which makes them less productive in the morning but more productive at night than they would have been in the absence of these devices.

We have found that just as those who utilize natural resources can influence the renewability of the resource with such things as fertilizer, the individual can also influence the renewability of alertness by influencing the rate at which it is acquired through sleeping. And just as those who mine natural resources have options to minimize its waste during extraction and utilization, so also does the individual have options to influence the rate at which alertness is lost during waking time. The economics of natural resources is thus a fruitful ground on which to build an economic analysis of sleep behavior.

Future work will involve the estimation of a demand curve for sleep, represented by equations (12) and (25). The existence of a downward sloping demand curve would also be in accordance with previous results (Biddle and Hamermesh 1990; Szalontai 2006). The empirical value of different parameters is of great interest, such as those governing the quality of sleep and the rate at which individuals lose alertness during the day. Multiple cross-price elasticities would be of interest and many of those concern developments that have raised great concern in the Western World in recent years. To give but one example, the substitutability between sleep duration and food consumption is both interesting as well as topical. This estimation could provide one of the missing puzzles regarding the causes of increases in obesity.

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